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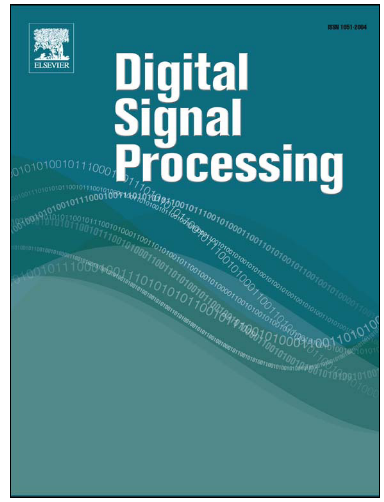

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Model-free fractional order differentiator based on fractional order Jacobi orthonormal functions

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Abstract

The aim of this paper is to design an algebraic and robust fractional order differentiator to estimate both the Riemann-Liouville and the Caputo fractional derivatives with an arbitrary order of an unknown signal in noisy environment, without knowing the model defining the signal. For this purpose, a new class of fractional order Jacobi orthonormal functions is firstly introduced. Secondly, the truncated fractional order Jacobi orthonormal series expansion is applied to filter the noisy signal, whose fractional derivative is used to estimate the desired one. Thus, the obtained differentiator is exactly given by an integral formula which depends on a set of design parameters. Thirdly, by applying the generalized Taylor's formula, some error analysis is provided. In particular, error bounds are given, which permit to study the design parameters' influence. Fourthly, a digital fractional order differentiator is deduced in discrete noisy case. Finally, by comparing with two existing fractional order differentiators, numerical results are given to illustrate the accuracy and the robustness of the proposed fractional order differentiator.

Key words: Fractional order differentiator, Fractional order Jacobi orthonormal functions, Error analysis.

1 Introduction

Fractional calculus was introduced in many fields of science and engineering long time ago. It was first developed by mathematicians in the middle of the ninetieth century [1]. During the past decades, fractional calculus has gained great interest in many scientific and engineering fields, including control, flow propagation, signal processing, image processing, electrical networks, and etc. [2–9]. In most cases, the fractional derivatives of a signal can not be analytically calculated. Moreover, when the signal is a unknown, it is usually measured in noisy environment. In order to estimate the fractional derivatives of an unknown signal from its discrete noisy observation, various robust fractional order differentiators have been designed in the frequency domain [10–12] and in the time domain [13–18]. Among them, there exists a class of model-free differentiators, which are obtained from the truncation of an analytical expression, without considering the model of the studied signal [10,11,13,14].

Recently, the model-free fractional order Jacobi differentiator was obtained by the fractional order differentiation by integration method [14], as the extension of the integer order one [19–23]. This method is algebraic, where the differentiator was exactly given by an integral formula. Moreover, it is robust against corrupting noises thanks to the integral [24]. The idea of this method is to filter the studied noisy signal by the Jacobi polynomial filter whose fractional derivative is used to approximate the one of the signal. Since the fractional derivative of a polynomial is a power function, the desired fractional derivative is approximated by a power function. Consequently, an intuitive idea is to approximate the studied signal by a power function whose fractional derivative is again a power function. Similar idea was used to solve fractional order differential equations in [25,26]. Bearing these ideas in mind, the aim of this paper is to extend the fractional order differentiation by integration method by introducing fractional order Jacobi orthonormal functions to filter the studied noisy signal, which

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