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Spatial smoothing based methods for direction-of-arrival estimation of coherent signals in nonuniform noise

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A B S T R A C T

Spatial smoothing techniques have been widely used to estimate the directions-of-arrival (DOAs) of coherent signals. However, in general these techniques are derived under the condition of uniform white noise and, therefore, their performance may be significantly deteriorated when nonuniform noise occurs. This motivates us to develop new methods for DOA estimation of coherent signals in nonuniform noise in this paper. In our methods, the noise covariance matrix is first directly or iteratively calculated from the array covariance matrix. Then, the noise component in the array covariance matrix is eliminated to achieve a noise-free array covariance matrix. By mitigating the effect of noise nonuniformity, conventional spatial smoothing techniques developed for uniform white noise can thus be employed to reconstruct a full-rank signal covariance matrix, which enables us to apply the subspace-based DOA estimation methods effectively. Simulation results demonstrate the effectiveness of the proposed methods.

1. Introduction

Direction-of-arrival (DOA) estimation using sensor arrays is an important task in many applications such as radar, sonar, and wireless communications. Usually, this problem is tackled by assuming uniform white noise, i.e., the noise covariance matrix is a scaled identity matrix. This assumption can reduce the number of unknown parameters and, therefore, the computational complexity [1]. In practice, the noise, however, could be colored [2–4] and non-Gaussian [5]. Particularly, in certain applications, the sensor noise is uncorrelated but variances across the array are not identical, which leads to the so-called nonuniform noise. In this case, DOA estimation approaches which rely on the assumption of uniform white noise cannot perform satisfactorily due to the incorrect noise model adopted [6].

Numerous studies have been devoted to the problem of DOA estimation in the presence of nonuniform noise. In [6], the maximum-likelihood (ML) estimator [1] for uniform noise has been extended to nonuniform noise through the stepwise concentration of the log-likelihood function with respect to the signal and noise nuisance parameters. Based on the similar scheme of stepwise concentration, a stochastic ML estimator is proposed in [7] for stochastic signals and an improved version of this algorithm has been reported in [8]. Madurasinghe [9] proposed a power domain approach which can relieve the computational burden of nonuniform ML estimators to some extent, since the DOA estimates are achieved by solving a nonlinear problem without iterations or determining the noise variances. In [10], a computationally attractive method which only needs a one-dimensional search is proposed. In [11], the noise covariance matrix is estimated by exploiting the relationship among the sub-matrices of the array covariance matrix. In [12,13], a noise-free sparse representation for DOA estimation is built by vectorizing the array covariance matrix and removing the entries which include noise variances. In [14], two optimization problems based on the ML and least-squares (LS) estimations are formulated to estimate the signal subspace and noise covariance matrix in an iterative manner. Unlike the nonuniform ML estimators, the unknown variables are obtained in an analytical form in each iteration. More recently, by assuming high signal-to-noise ratios (SNRs), improved subspace-based DOA estimators have been studied in [15].

It is worth noticing that the aforementioned methods are applicable to cases with uncorrelated signals only or noncoherent signals. As a matter of fact, even though algorithms such as [14] are theoretically able to handle any noncoherent signals, their performance would be significantly deteriorated when the signals are highly correlated [16]. To deal with coherent signals, numerous strategies using spatial smoothing [17–20] and higher-order statistics [21] have been proposed for uniform white noise and col-

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ored noise, respectively. However, the problem of DOA estimation of coherent signals in nonuniform noise has not been adequately addressed. To the best of our knowledge, only a generalized covariance differencing (GCD) approach based on the spatial smoothing principle has been proposed to eliminate nonuniform noise and resolve coherent signals [22]. However, in this approach, pseudo DOA estimates exist and have to be properly treated. Moreover, the algorithm is likely to break down when the pseudo and actual angles are not sufficiently separated.

Since spatial smoothing techniques are able to decorrelate coherent signals and reconstruct a full-rank signal covariance matrix, in this paper we propose to apply them to the noise-free covariance matrix, which is achieved by eliminating the noise covariance matrix from the array covariance matrix, to estimate the DOAs of coherent signals in nonuniform noise. To this end, two new noise covariance matrix estimation algorithms are devised. The first one directly calculates the noise covariance matrix from the array covariance matrix, while the other one extends the iterative least squares subspace estimation method [14] to the case of coherent signals. Simulation results show that the proposed methods can effectively resolve the coherent signals and provide outstanding DOA estimation performance.

2. Preliminaries

2.1. General signal model

Let us consider an M-element uniform linear array (ULA) receiving the signals emitted by L narrowband far-field sources with unknown DOAs $\{\theta_1, \cdots, \theta_L\}$. The number of sensor elements is larger than that of signals, i.e., $M > L$. At the tth time instant, the array observation consisting of the outputs of the sensors can be expressed as

$$\begin{aligned}\mathbf{x}(t) &= [x_1(t), x_2(t), \cdots, x_M(t)]^T = \sum_{l=1}^{L} \mathbf{a}(\theta_l) s_l(t) + \mathbf{n}(t) \\ &= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)\end{aligned} \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_L)] \in \mathbb{C}^{M\times L}$ denotes the steering matrix and $\mathbf{a}(\theta) \in \mathbb{C}^M$ denotes the steering vector as

$$\mathbf{a}(\theta) = [1, e^{j\frac{2\pi d}{\lambda}\sin\theta}, \cdots, e^{j\frac{2\pi d}{\lambda}(M-1)\sin\theta}]^T \tag{2}$$

where $(\cdot)^T$ denotes the transpose operation, d and λ are the inter-element spacing and carrier wavelength, respectively. $\mathbf{s}(t) = [s_1(t), \cdots, s_L(t)]^T \in \mathbb{C}^L$ and $\mathbf{n}(t) = [n_1(t), \cdots, n_M(t)]^T \in \mathbb{C}^M$ denote the signal and noise vectors, respectively, which are assumed to be uncorrelated. The signal covariance matrix is given by

$$\mathbf{P} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\} \in \mathbb{C}^{L\times L} \tag{3}$$

where $E\{\cdot\}$ and $(\cdot)^H$ represent the expectation and Hermitian transpose, respectively. The noise is nonuniform with covariance matrix

$$\mathbf{Q} = E\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \mathrm{diag}\{\boldsymbol{\sigma}\} \in \mathbb{R}^{M\times M} \tag{4}$$

where $\boldsymbol{\sigma} = [\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2]^T$, σ_m^2 is the noise variance of the mth sensor element, and diag$\{\cdot\}$ stands for a diagonal matrix composed of the bracketed elements. As a result, the array covariance matrix can be expressed as

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{R}_0 + \mathbf{Q} \tag{5}$$

where $\mathbf{R}_0 \triangleq \mathbf{A}\mathbf{P}\mathbf{A}^H$ denotes the noise-free covariance matrix. In practice, $\mathbf{R}$ can be estimated as $\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t)$ with N snapshots.

2.2. Coherent signals model

Assuming that the signals are coherent and $s_1(t)$ is the reference signal, the signal vector can thus be expressed as $\mathbf{s}(t) = [s_1(t), \alpha_2 s_1(t), \cdots, \alpha_L s_1(t)]^T$, where α_l, $l = 2, \cdots, L$, are nonzero complex-valued constants [17,18]. As a result, we have

$$\mathbf{x}(t) = s_1(t)\sum_{l=1}^{L}\alpha_l \mathbf{a}(\theta_l) + \mathbf{n}(t) = \mathbf{A}\boldsymbol{\alpha} s_1(t) + \mathbf{n}(t) \tag{6}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_M]^T$ with $\alpha_1 = 1$. Therefore, let us define $\mathbf{a}_0 \triangleq \mathbf{A}\boldsymbol{\alpha}$ and $p \triangleq E\{s_1(t)s_1^*(t)\}$, where $(\cdot)^*$ denotes the complex conjugate operator, then the array covariance matrix is expressed as

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = p\mathbf{a}_0\mathbf{a}_0^H + \mathbf{Q} = \mathbf{b}\mathbf{b}^H + \mathbf{Q} \tag{7}$$

where $\mathbf{b} = p^{1/2}\mathbf{a}_0$. In this case, the noise-free covariance matrix is given by $\mathbf{R}_0 = \mathbf{b}\mathbf{b}^H$. Since rank$\{\mathbf{R}_0\} = 1$, $\mathbf{b}$ can be interpreted as the scaled principal eigenvector of $\mathbf{R}_0$.

2.3. Generalized covariance differencing (GCD) algorithm

Let us divide the array into P overlapping subarrays, each of which has $\tilde{M}$ (where $\tilde{M} \geq L + 1$) sensor elements, then we have $P = M - \tilde{M} + 1$. For forward spatial smoothing, the kth subarray consists of the kth to $(k + \tilde{M} - 1)$th sensor elements, and the corresponding output vector is given by

$$\mathbf{x}_k(t) = [x_k(t), x_{k+1}(t), \cdots, x_{k+\tilde{M}-1}(t)]^T = \mathbf{F}_k\mathbf{x}(t) \in \mathbb{C}^{\tilde{M}} \tag{8}$$

where $\mathbf{F}_k = [\mathbf{0}_{\tilde{M}\times(k-1)}\ \ \mathbf{I}_{\tilde{M}}\ \ \mathbf{0}_{\tilde{M}\times(M-\tilde{M}-k+1)}] \in \mathbb{R}^{\tilde{M}\times M}$ is a selection matrix with $\mathbf{I}_{\tilde{M}}$ being the $\tilde{M}\times\tilde{M}$ identity matrix. The array covariance matrix associated with this subarray can thus be expressed as

$$\mathbf{R}_k^f = E\{\mathbf{x}_k(t)\mathbf{x}_k^H(t)\} = \mathbf{F}_k\mathbf{R}\mathbf{F}_k^H \in \mathbb{C}^{\tilde{M}\times\tilde{M}}. \tag{9}$$

As a result, the forward spatial smoothed covariance matrix is obtained as

$$\mathbf{R}^f = \frac{1}{P}\sum_{k=1}^{P}\mathbf{R}_k^f = \frac{1}{P}\sum_{k=1}^{P}\mathbf{F}_k\mathbf{R}\mathbf{F}_k^H. \tag{10}$$

For backward spatial smoothing, the sensor elements are numbered reversely, hence, the kth subarray consists of the $(k + \tilde{M} - 1)$th to kth sensor elements, and the complex conjugate of the output vector is given by

$$\mathbf{y}_k(t) = \mathbf{F}_k\mathbf{J}\mathbf{x}^*(t) \in \mathbb{C}^{\tilde{M}} \tag{11}$$

where $\mathbf{J} \in \mathbb{R}^{M\times M}$ denotes an exchange matrix with ones on the anti-diagonal and zeros elsewhere. Thus, we have

$$\mathbf{R}_k^b = E\{\mathbf{y}_k(t)\mathbf{y}_k^H\} = \mathbf{F}_k\mathbf{J}\mathbf{R}^*\mathbf{J}\mathbf{F}_k^H \tag{12}$$

and the backward spatial smoothed covariance matrix is computed as

$$\mathbf{R}^b = \frac{1}{P}\sum_{k=1}^{P}\mathbf{R}_k^b = \frac{1}{P}\sum_{k=1}^{P}\mathbf{F}_k\mathbf{J}\mathbf{R}^*\mathbf{J}\mathbf{F}_k^H. \tag{13}$$

Therefore, the forward/backward spatial smoothed covariance matrix $\mathbf{R}^{fb}$ can be obtained as

$$\mathbf{R}^{fb} = \frac{1}{2}(\mathbf{R}^f + \mathbf{R}^b) = \frac{1}{2P}\sum_{k=1}^{P}\mathbf{F}_k(\mathbf{R} + \mathbf{J}\mathbf{R}^*\mathbf{J})\mathbf{F}_k^H. \tag{14}$$

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