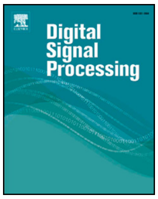
Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp



A factor graph-based iterative detection of faster-than-Nyquist signaling in the presence of phase noise and carrier frequency offset

Xiaotong Qi, Nan Wu *, Hua Wang, Weijie Yuan

School of Information and Electronics, Beijing Institute of Technology, Beijing, China

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Faster-than-Nyquist signaling
Phase noise (PHN)
Carrier frequency offset (CFO)
Factor graphs
Colored noise
Sum-product algorithm (SPA)
Variational message passing (VMP)

ABSTRACT

With the increasing demand for higher spectral efficiency in wireless communications, faster-than-Nyquist (FTN) signaling has been rediscovered to increase transmission rate without expanding signaling bandwidth. Most existing studies focus on low-complexity FTN receiver design by assuming perfect synchronization. In practice, however, phase noise (PHN) and carrier frequency offset (CFO) may degrade the performance of FTN detector significantly. In this paper, we develop iterative FTN detector in the presence of PHN and CFO in a factor graph framework. Wiener process is employed to model the time evolution of nonstationary channel phase. The colored noise imposed by sampling of FTN signaling is approximated by autoregressive model. Based on the factor graph constructed, messages are derived on the two subgraphs, i.e., PHN and CFO estimation subgraph and the FTN symbol detection subgraph. We propose two methods to update the messages between subgraphs, namely, Gaussian approximation via Kullback–Leibler divergence (KLD) minimization and the combined sum-product and variational message passing (SP-VMP), both of which enable low-complexity parametric message passing. The proposed SP-VMP algorithm can provide closed-form expressions for parameters updating. Moreover, conjugate gradient (CG) method is adopted to solve the maximum *a posteriori* probability (MAP) estimation of CFO with fast convergence speed. Simulation results show the superior performance of the proposed algorithm compared with the existing methods and verify the advantage of FTN signaling compared with the Nyquist counterpart.

1. Introduction

Faster-than-Nyquist (FTN) signaling, first proposed by Mazo in 1970s [1], has been rediscovered in recent years as a promising technique to improve spectral efficiency while preserving the signaling bandwidth [2–4]. It has been proved that FTN signaling can attain the same asymptotic error probability as the Nyquist signaling as long as the packing factor is above the Mazo limit [5]. Nevertheless, since the shaping pulses of FTN signaling are packed to transmit more symbols during the same time interval T, the intersymbol interference (ISI) is unavoidable.

Many studies have been performed to eliminate the intentional ISI imposed by FTN [6–11]. In [6], the FTN signaling is considered as a convolutional code and Viterbi algorithm is applied for symbol detection. A maximum *a posteriori* probability (MAP) detector with successive interference cancellation (SIC) is employed in [7]. In [8], a reduced-complexity turbo equalization-based M-algorithm BCJR (M-BCJR) algorithms have been proposed. A frequency-domain equalization (FDE)-based FTN receiver is studied in [9], where the impact of ISI of FTN signaling is approximated by circulant matrix structure, which enables low-complexity minimum mean squared error (MMSE) symbol detection. The FDE-aided iterative detector is employed in a three-stage-concatenated FTN system in [10] to further improve the performance. However, the colored noise imposed by sampling the FTN signaling is not fully considered in [9, 10]. Using autoregressive (AR) process to model the colored noise, a graph-based linear MMSE (LMMSE) equalizer is developed in [11].

The existing studies on receiver design of FTN signaling assume perfect synchronization. In practice, however, local oscillator instabilities of both transmitter and receiver will lead to phase noise (PHN), which becomes one of the major impairments to the performance of communication systems. It is further exacerbated by the recent demand for larger bandwidth at higher frequencies, where PHN becomes much more severe as the oscillator frequency increases. There have been many researches to address the impact of PHN for Nyquist signaling [12–17]. In [14] an iterative decoding algorithm for channels impacted by strong PHN is proposed,

---

* Corresponding author.
E-mail addresses: xtseven@bit.edu.cn (X. Qi), wunan@bit.edu.cn (N. Wu), wanghua@bit.edu.cn (H. Wang), wjyuan@bit.edu.cn (W. Yuan).

http://dx.doi.org/10.1016/j.dsp.2016.12.011

Please cite this article in press as: X. Qi et al., A factor graph-based iterative detection of faster-than-Nyquist signaling in the presence of phase noise and carrier frequency offset, Digit. Signal Process. (2016), http://dx.doi.org/10.1016/j.dsp.2016.12.011

which utilizes the sum-product algorithm (SPA) [18] to derive the MAP symbol detector. Canonical distribution approach [19] has been employed to represent the probability density function (pdf) of PHN by Tikhonov distribution. In [15], variational bounding is employed to design the iterative detection algorithm, and then Taylor-series expansion is used to linearize the system model and extended Kalman smoothing is applied to calculate the soft estimation of PHN. Based on variational inference, [16] constraints the free distribution as Gaussian or Dirac delta function to further reduce the complexity. In [17], by partitioning the received frame into partial blocks, the PHN is estimated based on discrete cosine transform (DCT) expansion. The estimation accuracy of the above algorithms degrades rapidly in the presence of carrier frequency offset (CFO). Although many pilot-aided CFO estimators can be employed [20], long training sequence leads to spectral efficiency loss. On the other hand, the presence of strong PHN may also degrade the accuracy of CFO estimation. Therefore, joint estimation of CFO and PHN should be considered. In [21], a joint detection and decoding algorithm in the presence of PHN and CFO is proposed based on factor graph, where CFO is quantized with equally spaced level and processed in parallel for each quantization value.

In this paper, we develop an iterative detection algorithm for FTN signaling in the presence of PHN and CFO. The impact of PHN and CFO is modeled as Wiener process and the colored noise imposed by the sampling of FTN signaling is approximated by AR model. Building on this, the joint posterior distribution is factorized efficiently and the corresponding factor graph is constructed. For ease of exposition, we further divide the whole factor graph into two subgraphs, namely, PHN and CFO estimation and FTN symbol detection, and the updating rules of messages are derived on each subgraphs. The proposed algorithm results in low-complexity parametric message passing to detect FTN symbols iteratively on factor graph. Simulations results demonstrate the superior performance of the proposed algorithm in FTN system compared with the state-of-the-art methods and the Nyquist counterpart. The main contributions of this paper are summarized in the following:

- Different from existing works that assume perfect synchronization, we study more practical situation of FTN signaling detection in the presence of PHN and CFO. Based on the efficient factorization of the joint posterior distribution, we solve the joint detection and estimation problem in factor graph framework.
- We propose two methods to update the messages between subgraphs, i.e., Gaussian approximation via Kullback–Leibler divergence (KLD) minimization and the combined sum-product and variational message passing (SP-VMP). Both of the two methods enable us to perform parametric message passing on factor graph. Especially, the proposed SP-VMP algorithm results in closed-form parameters updating rules.
- Instead of performing quantization on CFO as in [21], we use Dirac delta function with the MAP estimate to represent the message from the variable node of CFO, which avoids the complex parallel processing for each quantized CFO value in [21]. Conjugate gradient (CG) method is adopted to solve the optimization with faster convergence speed.

The remainder of this paper is organized as follows. Section 2 describes the system model. In Section 3, the probabilistic model of FTN signaling detection is presented and the factor graph is constructed. The updating rules of messages are derived on factor graph. The simulation results and discussions are given in Section 4. Finally, conclusions are drawn in Section 5.

Notations: We use boldface capital letter to denote a matrix while boldface lower-case letter for a vector. $\Re(\cdot)$, $(\cdot)^*$ represent the real part, complex conjugate of a complex number; $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^{-1}$ are the transpose, conjugate transpose and the inverse operator, respectively; $g(m_x, v_x; x)$ and $g_{\mathcal{C}}(m_x, v_x; x)$ are real and complex Gaussian distribution of variable x with mean m_x and variance v_x, respectively; $\propto$ represents equality up to a constant normalization factor; $\mathbf{W}$ is the weight matrix; $\mathcal{S}\backslash s$ denotes all elements in the set $\mathcal{S}$ but s.

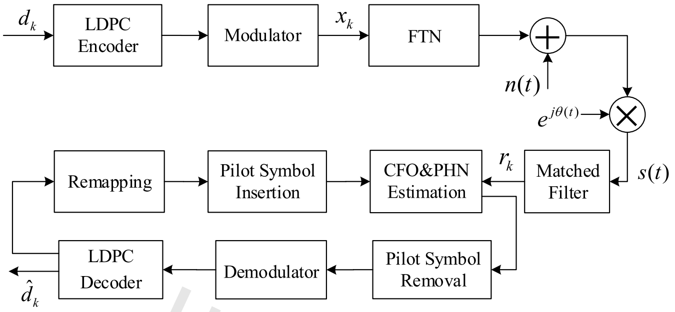


Fig. 1. The block diagram of FTN system.

2. System model

We consider a low-density parity-check (LDPC)-coded linearly modulated FTN system illustrated in Fig. 1. The mapping function from information bits $\mathbf{d} = [d_0, \dots, d_{N-1}]^T$ to coded symbols $\mathbf{x} = [x_1, \dots, x_K]^T$ is denoted by $\mathbf{x} = \zeta_{\mathcal{C}}(\mathbf{d})$. The symbol sequence $\mathbf{x}$ is passed through a T-orthogonal root-raised-cosine (rRC) pulse shaping filter $g(t)$ with signaling rate $\frac{1}{\tau T}$, where $0 < \tau \le 1$ is the packing factor of FTN signaling and T is the symbol interval of the Nyquist signaling. Then, the signal is transmitted over an additive white Gaussian noise (AWGN) channel. At receiver side, the signal is affected by PHN and CFO, which is given by

$$s(t) = [\sum_m x_m g(t - n\tau T) + n(t)]e^{j\theta(t)}, \tag{1}$$

where $\theta(t)$ represents the time varying channel phase and $n(t)$ is an additive white circularly symmetric complex Gaussian noise with power spectral density N_0. Assuming perfect symbol timing between the transmitter and receiver, the received signal is passed though the matched filter and sampled with period τT. The signal sample at the kth time instance is

$$r_k = \int_{-\infty}^{+\infty} s(t) g^*(t - k\tau T)dt. \tag{2}$$

Substituting (1) into (2) yields

$$r_k = c_k e^{j\theta_k} + \omega_k, \tag{3}$$

where $c_k = \sum_{m=-\infty}^{\infty} x_m h_{m-k}$ represents the symbols affected by the FTN imposed ISI with $h_{m-k} = \int_{-\infty}^{+\infty} g(t - m\tau T)g^*(t - k\tau T)dt$, ω_k is the colored noise samples with autocorrelation function given as [22]

$$R(m-k) = \mathbb{E}[\omega_m \omega_k^*] = N_0 h_{m-k}. \tag{4}$$

We can use autoregressive model of the first order to approximate the colored noise process, which is expressed as

$$\omega_k = \alpha\omega_{k-1} + \epsilon_k, \tag{5}$$

where $p(\epsilon_k) = g_{\mathcal{C}}(0, \sigma_\epsilon^2; \epsilon_k)$, the parameters α and σ_ϵ^2 can be obtained via Yule–Walker equation [23].

The time evolution of the channel phase due to PHN and CFO can be modeled by Wiener process. The discrete-time sample of $\theta(t)$ follows

Please cite this article in press as: X. Qi et al., A factor graph-based iterative detection of faster-than-Nyquist signaling in the presence of phase noise and carrier frequency offset, Digit. Signal Process. (2016), http://dx.doi.org/10.1016/j.dsp.2016.12.011