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Array geometry impact on music in the presence of spatially distributed sources

Xiong Wenmeng *, Picheral José, Marcos Sylvie

Laboratoire des signaux et systèmes, CentraleSupélec–CNRS–Université Paris-Sud, 91192 Gif-sur-Yvette, France

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The MUltiple SIgnal Classification (MUSIC) estimator has been widely studied for a long time for its high resolution capability in the domain of the direction of arrival (DOA) estimation, with the sources assumed to be point. However, when the actual sources are spatially distributed with angular dispersion, the performance of the conventional MUSIC is degraded. In this paper, the impact of the array geometry on the DOA estimation of spatially distributed sources impinging on a sensor array is considered. Taking into account a coherently distributed source model, we establish closed-form expressions of the MUSIC-based DOA estimation error as a function of the positions of the array sensors in the presence of model errors due to the angular dispersion of the signal sources. The impact of the array geometry is studied and particular array designs are proposed to make DOA estimation more robust to source dispersion. The analytical results are validated by numerical simulations.[1]

1. Introduction

The DOA estimation based on snapshots received on a sensor array has been widely studied with plenty of methods [2]. Among these methods, the MUltiple SIgnal Classification (MUSIC) [3] is famous for its high resolution in the case of point sources. However, in many applications, such as acoustic source imaging [4] and mobile channel communication [5], where angular dispersion of the sources up to 10° may occur, the physical sources can no longer be considered as points. In this case the performances of the DOA estimation obtained by the conventional MUSIC are degraded, and a spatially distributed model of the sources would be more appropriate.

The models for spatially distributed sources have been classified into two types, namely incoherently distributed (ID) sources and coherently distributed (CD) sources [6]. On one hand, for ID sources, signals coming from different points of the same distributed source can be considered uncorrelated. On the other hand, in the scenario of CD sources, the received signal components are delayed and scaled replicas from different points of the same one [6]. For CD sources, the performances of MUSIC with discretely distributed sources and continuously distributed sources have been investigated in [7], and [8], respectively. As expected, due to the angular dispersion the mismatch between the steering vector model of MUSIC and the actual steering vectors of the sources causes estimation errors. Plenty methods such as the joint estimation of both the DOA and the angular dispersion parameter [6,9,10] have been proposed to solve these problems. However, these methods require a high computational burden or a knowledge of the shape of the source angular distribution for the model, which motivates us to rather keep the conventional point-source MUSIC for the DOA estimation and seek other ways to improve the performances as, for example, the optimization of the array geometry.

The array geometry effect on the DOA estimation has been studied in plenty of publications and in different contexts. The uniform linear array (ULA) is the simplest. However, a more complex geometry can lead to better performance. Optimal array geometries have been designed to reach isotropic and/or optimal performance based on the Cramér–Rao bound (CRB) criterion (e.g.: [11–13]), the lower bound of the mean square error (MSE) can be uniform for all the possible DOAs, or the inferior bound of the MSE of the DOA in the elevational and horizontal direction can be decoupled. More recently, based on the spatial aliasing phenomenon, a class of non-uniform array geometries composed of two or more uniform linear arrays (ULAs) with different inter-element spacing has been used to reduce the computational burden of the Maximum Likelihood (ML) estimator [14]. Also, many techniques have been applied in the sparsity array design or large-scale broad-

* Corresponding author.
E-mail address: xiongwenmeng@outlook.fr (W. Xiong).

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band array to reduce the number of elements in an array, to offer a lower cost, power consumption, and heat dissipation (e.g.: [15–17]).

In our work, we focus on the impact of the array geometry on the performance of the MUSIC estimator with the CD source model, in the presence of errors due to modeling mismatch of source dispersion. Let us first briefly review our work in [8] and [1]. In [8], the point source MUSIC has been extended into CD-MUSIC (coherent source MUSIC), the first order Taylor approximation DOA estimation bias is proposed in the case that the steering vectors in the model and of the actual sources are mismatched. In [1] we make use of the theoretical estimation bias expression in [8], and express the DOA estimation bias as a function of the sensor positions. Impacts of particular array geometries on the estimator performances are studied in the scenario of single source and multiple sources, respectively. In this paper, besides the results in the conference papers, our contributions consist of: firstly, a general condition for canceling the DOA estimation bias in the case of one source is derived; secondly, the condition for canceling the cross terms of the DOA and the angular dispersion parameter in Cramér–Rao bound in the case of one source is derived; thirdly, for the results concerning UCA in the case of two sources in [1], the calculation details are given in 6.1.

The organization of this paper is as follows. The signal model and a brief recall of MUSIC are given in section 2. The impact of array geometry on the performance of MUSIC and on the crossed terms of the CRB are studied in section 4. Particular geometry designs are studied in section 5. UCA for the case of two sources is studied in section 6. Finally, conclusions are given in section 7.

2. Signal model

Let us consider q spatially CD far-field narrow-band sources impinging on an array of M sensors. The sources arrive from the DOA $\theta_1, \ldots, \theta_q$, and the position of the m-th sensor is given by the polar coordinates ρ_m and α_m. Without loss of generality, the signals and the sensors are assumed to be in the same plane, as shown in Fig. 1. The q source signals and the M signals received by the array at moment t are denoted by $\mathbf{s}(t) = [s_1(t), \ldots, s_q(t)]^T$ and $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T$, respectively. In the case of CD sources, it is common to exploit the model proposed in [6] in order to express the received signal:

$$\mathbf{y}(t) = \mathbf{C}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ represents the complex Gaussian distributed additive noise, $\mathbf{C}(\theta) = [\mathbf{c}_{h_1}(\theta_1), \ldots, \mathbf{c}_{h_q}(\theta_q)] \in \mathbb{C}^{M\times q}$ is the array steering matrix composed of q steering vectors $\mathbf{c}_{h_i}(\theta)$ that can be written by:

$$\mathbf{c}_{h_i}(\theta_i) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{a}(\theta_i + \phi) h_i(\phi) d\phi, \tag{2}$$

where $i = 1 \ldots q$, and $\mathbf{a}(\theta)$ is the steering vector for a point source, which can be given by:

$$\mathbf{a}(\theta_i) = \left[e^{-j2\pi \frac{\rho_1}{\lambda}\cos(\theta_i - \alpha_1)}, \ldots, e^{-j2\pi \frac{\rho_M}{\lambda}\cos(\theta_i - \alpha_M)} \right]^T, \tag{3}$$

where λ is the wavelength, and $[\cdot]^T$ is the transpose operation.

The functions $h_i(\phi)$ are introduced in (2) to describe the angular spread distribution (for instance, Uniform and Gaussian distributions). The source signals and the additive noise are considered to be complex centered Gaussian independent random variables. Assuming that signals and noises are uncorrelated and the sources are uncorrelated with each other, the correlation matrix is given by:

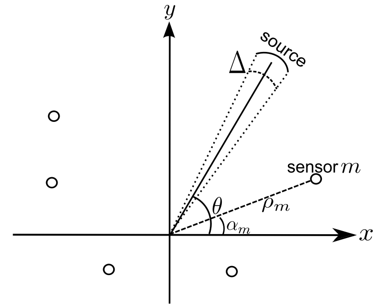

Fig. 1. Planar array and source DOAs.

$$\mathbf{R} = E[\mathbf{y}\mathbf{y}^H] = \mathbf{C}\mathbf{R}_s\mathbf{C}^H + \sigma_b^2\mathbf{I}, \tag{4}$$

where $E[.]$ is the expectation operator, $\mathbf{R}_s$ and σ_b^2 are the source covariance matrix and the noise variance, respectively.

Under the hypothesis that $q < M$ and $\mathbf{R}_s$ and $\mathbf{C}$ are not rank deficient, it is well known that the decomposition of $\mathbf{R}$ into eigenvalues λ_m and eigenvectors $\mathbf{e}_m$ is as follows:

$$\mathbf{R} = \sum_{m=1}^{M} \lambda_m \mathbf{e}_m \mathbf{e}_m = \mathbf{U}\Lambda_s\mathbf{U}^H + \sigma_b^2\mathbf{V}\mathbf{V}^H, \tag{5}$$

where $\mathbf{U} = [\mathbf{e}_1, \ldots, \mathbf{e}_q]$ spans the signal subspace defined by the columns of $\mathbf{C}$, $\mathbf{V} = [\mathbf{e}_{q+1}, \ldots, \mathbf{e}_M]$ spans the noise subspace defined as the orthogonal complement of $\mathbf{U}$, and $\Lambda_s = diag\{\lambda_1, \ldots, \lambda_q\}$.

3. MUSIC estimator and performances

The MUSIC [3] method makes use of the orthogonal property of the subspaces spanned by $\mathbf{C}(\theta)$ and $\mathbf{V}$ to estimate the DOAs θ_i. In practice it is difficult to know exactly the angular dispersion of the actual sources, consequently, the steering vector model of the point source $\mathbf{a}(\theta)$ is used here instead of $\mathbf{c}_{h_i}(\theta)$ to estimate the value of θ:

$$\hat{\theta}_i = \underset{\theta}{\operatorname{argmax}} \frac{1}{\|\mathbf{a}^H(\theta)\mathbf{V}\|^2}. \tag{6}$$

Here, we assume that the number of snapshots is large enough such that the estimation error of the noise subspace can be neglected and the DOA estimation error comes mainly from the model error, it is to say, the mismatch of the angular dispersion parameter between the model of the MUSIC estimator $\mathbf{a}(\theta)$ and the actual source $\mathbf{c}_{h_i}(\theta_i)$. We recall here the standard analysis in order to express the estimation error $\Delta\theta_i = \hat{\theta}_i - \theta_i$, $\hat{\theta}_i$ should satisfy that the first derivative of denominator in (6) is null:

$$\left.\frac{\partial \mathbf{a}(\theta)^H \mathbf{V}\mathbf{V}^H \mathbf{a}(\theta)}{\partial \theta}\right|_{\hat{\theta}_i} = 0, \tag{7}$$

which gives:

$$2\mathcal{R}e\{\dot{\mathbf{a}}(\hat{\theta}_i)^H \mathbf{V}\mathbf{V}^H \mathbf{a}(\hat{\theta}_i)\} = 0, \tag{8}$$

where $\dot{\mathbf{a}}(\hat{\theta}_i) = \left.\frac{\partial \mathbf{a}(\theta)}{\partial \theta}\right|_{\hat{\theta}_i}$.

Assuming that, $\hat{\theta}_i$ is not far away from θ_i, we make the first order approximation of Taylor:

$$\mathbf{a}(\hat{\theta}_i) \approx \mathbf{a}(\theta_i) + \Delta\theta_i \dot{\mathbf{a}}(\theta_i), \tag{9}$$

and:

$$\dot{\mathbf{a}}(\hat{\theta}_i) \approx \dot{\mathbf{a}}(\theta_i) + \Delta\theta_i \ddot{\mathbf{a}}(\theta_i), \tag{10}$$

where $\ddot{\mathbf{a}}(\theta_i) = \left.\frac{\partial \dot{\mathbf{a}}(\theta)}{\partial \theta}\right|_{\theta_i}$.