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Low complexity performance assessment of a sensor array via unscented transformation

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ABSTRACT

Due to the advances on electronics, applications of antenna array signal processing are becoming more frequent. When employing antenna arrays for beamforming, the signal to interference and noise ratio (SINR) should be assessed. Many factors can affect the SINR such as the array element positioning error and the direction of arrival (DOA) estimation error. In these cases, the assessment is traditionally performed via the SINR average obtained using Monte Carlo (MC) simulations. However, this approach requires a great amount of realizations that demand a high computational effort and processing time due to its slow convergence. In this paper, we propose a low complexity performance assessment of the average SINR via unscented transformation. Compared to MC simulations, our proposed method requires only a few trials and has a negligible computational complexity, yet giving a comparable SINR when the DOA estimation is perturbed. When the antenna elements positioning is perturbed, a multivariate scenario arises. For multivariate scenario the proposed scheme has an exponential increase in complexity, therefore, still being advantageous for a small number of antennas.

1. Introduction

Applications such as speech and audio acquisition [1,2], wireless communications [3] and RADAR [4] make use of array signal processing in order to enhance their capabilities. One of the most common uses of antenna arrays is spatial filtering by the use of beamformers [5]. However, idealistic assumptions such as a known direction of arrival (DOA) of the desired signal or perfectly spaced array elements are usually made [6]. Therefore, a performance assessment in the presence of deviations should be considered for practical implementations.

Geometry based beamformers, e.g. delay and sum (DS), generalized sidelobe cancellers (GSC) and minimum variance distortionless response (MVDR), take one or more DOAs as input parameters of the associated optimization problem. Though, DOA estimation is always prone to a certain degree of error. Moreover, the positioning of the antenna elements is not always perfectly known and it may affect the beamformer's quality. In this paper, the quality is measured as the average of the achieved signal to interference and noise ratio (SINR). The average is important, since the random nature of these perturbations will lead to random SINR values that may cause inconclusive results. For example, a simulation in a more favorable scenario may result in an SINR that is lower than that of a simulation in a less favorable one. However, these values vary around a mean and computing the average gives the system designer an overall SINR, i.e. which SINR level is expected when the system is subject to a certain degree of error.

The Monte Carlo (MC) method [7] is a commonly used simulation technique for the computation of the average SINR [8] due to its simplicity and easiness of implementation. However, it requires a large number of trials [9] to converge to a satisfactory result, implying a long simulation time. Currently, performance assessment of embedded systems takes 30% of the development time and it could increase to 70% [10]. Therefore, improving the efficiency of performance assessment tools implies reducing production costs and delivering new solutions faster to the market.

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Previous works derive analytical expressions to assess the system's quality by using first order expansions of the perturbed parameters [11,12]. These analytical expressions evaluate the perturbation due to noise and are exact for high signal to noise ratio (SNR) values, but they do not present a good fit for low SNR cases. In this paper, we study the effect of other type of perturbations, more precisely the DOA estimation error and antenna element positioning error on the SINR. We show that, in these cases, the computation of such analytical expressions is hard or not practical and we propose the use of the unscented transformation (UT) to numerically evaluate the SINR. The UT maps a continuous probability distribution into a discrete one with the same statistical moments [13]. When a non-linear function is applied to the mapped distribution, in our case the SINR function, the results give us a good fit in comparison to the traditional MC approach, yet with a negligible computational time for univariate perturbation models. When the perturbation is multivariate, e.g. error in the position of each antenna element, the complexity grows exponentially with the number of antennas. Therefore, the complexity is still lower than the MC method's complexity for a small number of antennas and greater for a large number of antennas. In order to alleviate the effect of the array spacing perturbation using the UT, the reader is referred to [14].

In this work, two perturbations are considered, a DOA estimation error and array element positioning error. We assume uncorrelated and equipowered sources so that the source signal covariance is an identity matrix and the covariance matrix can be computed as explained in Section 4. The evaluation of these perturbations gives raise to a univariate and a multivariate UT, respectively. For the sake of demonstration, one type of perturbation is considered for each case. In future work, the UT may also be applied for other types of perturbations such as frequency shift, mutual coupling, amplitude error and phase error. Also, other integration methods such as the quadrature and cubature transforms [15] showed better results than the UT for filtering purposes. Even though the quadrature and cubature transforms might also be considered for sensor array performance assessment, we regard them as future work and focus on the simplicity and ease of implementation of the UT.

The remainder of this paper is divided as follows. Section 2 shows the data model containing the considered perturbations. In Section 3, we overview basic concepts of the UT. In Section 4, we propose the performance assessment of a beamformer using the UT. Section 5 presents the simulations results. Finally, Section 6 draws the conclusions of this work.

2. Data model

We start with the received signal model of an ideal uniform linear array (ULA) containing M antenna elements

$$\mathbf{x}(t) = \sum_i \mathbf{a}(\theta_i)s_i(t) + \mathbf{v}(t) \in \mathbb{C}^{M\times 1}, \tag{1}$$

where $\mathbf{a}(\theta_i)$ is the steering vector, θ_i is the DOA azimuth of i-th source signal $s_i(t)$ and $\mathbf{v}(t)$ is the additive noise term. Since we are dealing with a ULA, the steering vector contains the phase delays $a_{m,i} = e^{j2\pi\mu_i}$, where $\mu_i = (m-1)\frac{d}{\lambda}\cos\theta_i$, m is the antenna element index and d is the inter-element spacing in wavelengths. If d is given in wavelengths, the wavelength can be dropped from the phase delay μ_i.

Let us consider the case where DOA and element positioning errors are present as depicted in Fig. 1.

Fig. 1 shows that a plane wavefront reaches the ULA with a DOA angle θ_i and is subject to an additive deviation represented by the Gaussian distributed random variable $\Theta \sim \mathcal{N}(0, \sigma_\theta^2)$. Therefore, the received signal in the presence of DOA estimation error becomes:

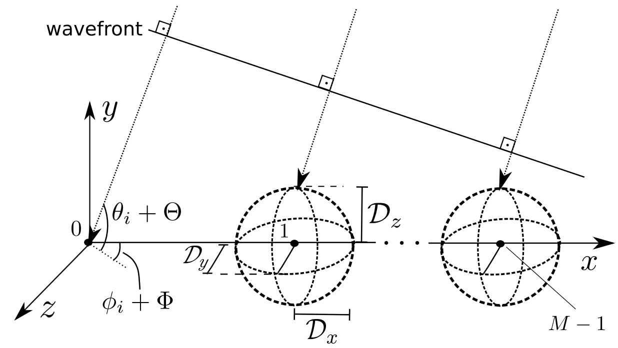

Fig. 1. Illustration of a plane wavefront impinging on an antenna array containing DOA and element positioning errors.

$$\mathbf{x}^{(\Theta)}(t) = \mathbf{a}^{(\Theta)}(\theta_0)s_0(t) + \sum_{i\neq 0} \mathbf{a}(\theta_i)s_i(t) + \mathbf{v}(t) \in \mathbb{C}^{M\times 1}, \tag{2}$$

where $\mathbf{a}^{(\Theta)}(\theta_0) = \mathbf{a}(\theta_0 + \Theta)$ is the steering vector perturbed by a random variable Θ and the signal of interest (SOI) corresponds to the $i = 0$ source signal. Note that (2) models the DOA estimation error as a physical change in the signal's direction.

Also, in Fig. 1, the solid dots represent the antenna element positions. The first element is the reference of the Cartesian axes. The remaining elements are positioned at $(m-1)d$ along the x axis. Each of the elements, except the reference element, is subject to a positioning error in all of the 3 space dimensions x, y and z and are modeled by the random variables $\mathcal{D}_x$, $\mathcal{D}_y$ and $\mathcal{D}_z$, respectively.

When the three dimensions are considered, not only the azimuth θ_i of the DOA, but also its elevation ϕ_i matters. Expanding the phase delays for the three dimensions we obtain

$$\begin{aligned}\mu_{m,i} &= (m-1)d\cos\theta_i\cos\phi_i + \mathcal{D}_{x,m}\cos\theta_i\cos\phi_i \\ &\quad + \mathcal{D}_{y,m}\sin\theta_i\cos\phi_i + \mathcal{D}_{z,m}\sin\phi_i,\end{aligned} \tag{3}$$

where the subscript m indicates the antenna index, since the random variables are independent with respect to each other.

The received signal for a random array positioning can be written as

$$\mathbf{x}^{(\boldsymbol{\mathcal{D}})}(t) = \sum_i \mathbf{a}^{(\boldsymbol{\mathcal{D}})}(\theta_i, \phi_i)s_i(t) + \mathbf{v}(t) \in \mathbb{C}^{M\times 1}, \tag{4}$$

where $\mathbf{a}^{(\boldsymbol{\mathcal{D}})}(\theta_i, \phi_i) = [e^{j\mu_{1,i}}, e^{j\mu_{2,i}}, \ldots, e^{j\mu_{M,i}}]^{\mathrm{T}}$ is the steering vector perturbed by the random vector $\boldsymbol{\mathcal{D}} = [\mathcal{D}_{x,1}, \mathcal{D}_{y,1}, \mathcal{D}_{z,1}, \mathcal{D}_{x,2}, \ldots, \mathcal{D}_{z,M}]$.

3. Unscented transformation

The Unscented Transformation (UT) is based on the mapping of a continuous probability distribution into a discrete one and can be used to compute the moments of non-linear transformations of a random variable [16]. Traditionally, such moments are computed via MC simulations. However, this approach requires large computational efforts and, depending on the accuracy, the computational complexity can be prohibitive.

In this section we review the concepts of the UT. The UT for a single random variable is reviewed in Section 3.1 and its extension for multiple i.i.d. random variables is reviewed in Section 3.2, respectively.