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Nonlinear regression A*OMP for compressive sensing signal reconstruction

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ABSTRACT

A number of tree search based methods have recently been utilized for compressive sensing signal reconstruction. Among these methods, a heuristic algorithm named A* orthogonal matching pursuit (A*OMP) follows best-first search principle and employs dynamic cost model which makes sparse reconstruction exceptionally excellent. Since the algorithm performance of A*OMP relies heavily on preset parameters in the cost model and the estimation of these preset parameters requires a large number of experiments, there is room for improvement in A*OMP. In this paper, an improved algorithm referred to as Nonlinear Regression A*OMP (NR-A*OMP) is proposed which is built on the residue trend to avoid the estimation procedure. This method is inspired by the fact that the residue is correlated closely to the measurement matrix. The residue trend reflects the characteristics of nonlinear regression with the increasing of sparsity K. In addition, restricted isometry property (RIP) based general conditions are introduced to ensure the effectiveness and practicality of the algorithm. Numerical simulations demonstrate the superiority of NR-A*OMP in both reconstruction rate and normalized mean squared error. Results indicate that the performance of NR-A*OMP can become nearly equal to or even better than that of A*OMP with perfect preset parameters.

1. Introduction

Traditional approaches to capturing and reconstructing signals follow the Shannon/Nyquist sampling theorem, which states that the sampling rate must be at least twice the maximum frequency component presented in the signal to avoid information loss. This fundamental principle constitutes the basis of many other signal acquisition protocols which have been widely accepted in practical applications, including wireless communication, medical imaging, and so on.

In the last decade, a rapid developing theory, which goes the name by compressive sensing or compressed sensing [1–4] has provided a second choice beyond the Shannon/Nyquist law when extra conditions meet. With its help, sparse or compressible signals can be captured and represented at an extremely low sampling rate that falls far below the supposed sampling rate recommended by the Shannon/Nyquist theorem. More surprisingly, the paradigm suggested by compressive sensing underlies procedures for sampling and compressing data simultaneously, which overcomes common wisdom in traditional data acquisition.

Although compressive sensing seems to be a very promising mathematical tool to reduce both relative data storage and calculation complexity, an intractable problem needs to be handled before it finally comes into practical application. For instance, the classic compressive sensing theory, which is related with a minimum ℓ_0-norm problem, requires an exhaustive combinatorial search within the framework of compressive sensing, thus resulting in an NP-hard problem.

To solve this problem, a variety of strategies have been emerged and could be mainly classified into three categories, namely greedy algorithms [5–9], convex optimization algorithms [10–12] and combinatorial algorithms [13–15]. Generally, convex optimization algorithms outperform combinatorial algorithms in computational accuracy but have the shortcoming of expensive computation. Greedy algorithms have merits in both of computational accuracy and computational complexity owing to reaching a good compromise between convex optimization algorithms and combinatorial algorithms. Moreover, greedy algorithms have some benefit from the high flexibility, which leads to easy integration with various mathematical methods to improve algorithm performance effectively. For example, iterative shrinkage/thresholding algorithms (ISTA), as an important branch of greedy algorithms, developed from gradient descent method have received a consid-

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erable amount of attention in the field of medical imaging [16–18]. ISTA avoid computing the inverse of the high-dimensional matrix to improve the real-time performance compared to the other algorithms.

In the line of greedy algorithms, orthogonal matching pursuit (OMP) [5] is the most classic and basic one that has attracted much attention in the research community. Although it has been proved, simple structure and low complexity, its reconstruction rate (RR) still needs to be further improved. The regularized orthogonal matching pursuit (ROMP) [6] is presented to partly deal with this problem by combining greedy algorithms with convex optimization and its modified version termed as the compressive sampling matching pursuit (CoSaMP) [7], guarantees this improvement by employing a pruning step. However, their performances in the noisy scenario are far from being satisfactory. Multipath matching pursuit (MMP) [19] is proposed to fit for more complex situations by introducing tree search from graph theory, but it is rather an unsophisticated technique that explores a search tree following predefined orders, including Breadth-First-Search and Depth-First-Search. Inspired by MMP, A* orthogonal matching pursuit (A*OMP) [20–23] employs an intelligent selection procedure according to a classic pathfinding method termed A* search and performs better accuracy and flexibility.

However, the good performance of A*OMP is at the cost of complex mathematical models and accurate preset parameters selection based on data training. Otherwise, the degradation and deterioration are foreseeable. To enhance the robustness to the lack of a priori knowledge, we propose a novel approach referred to as Nonlinear Regression A*OMP (NR-A*OMP) that investigates the residue trend when the signal is reconstructed. For this purpose, we study the correlation between the residue and the measurement matrix to match the desired solution to NR-A*OMP, then survey and provide the crucial mathematical insights underlying this new method.

The rest of this paper is organized as follows. In Section 2, the framework of compressive sensing is given, greedy algorithms and the semi-greed algorithm are introduced briefly. In Section 3, NR-A*OMP is proposed to overcome the disadvantage of the original algorithm, including algorithmic idea, mathematical expression, flow chart, and the residue trend. In Section 4, the RIP conditions for NR-A*OMP are presented to ensure the perfect recovery of sparse signals. In Section 5, numerical simulations are provided to demonstrate the residue trend and the effectiveness of the proposed method. Finally, some concluding remarks are drawn in Section 6.

2. Background

2.1. Compressive sensing

If a vector is sparse or, more generally, compressible in some basis, a condensed representation could be obtained directly. Through non-adaptive linear projections, the information loss could be ignored. Then the original signal could be reconstructed via an optimization method with high probability, which could be mathematically depicted by

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^M$ is the measurement vector, $\mathbf{x} \in \mathbb{R}^N$ is the original vector, $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is the measurement matrix or called redundant dictionary, the columns of $\mathbf{\Phi}$ are dictionary atoms. The top concern we care about is the undersampled case when $M < N$. Obviously, (1) itself could not be used to recover $\mathbf{x}$ due to underdetermination, which could only work under the premise that vector $\mathbf{x}$ is K-sparse, i.e. $\|\mathbf{x}\|_0 \le K$. More specifically, the ill-posed problem is radically formulated as following optimization model.

$$\hat{\mathbf{x}} = \operatorname{argmin} \|\mathbf{x}\|_0 \quad s.t. \quad \mathbf{y} = \mathbf{\Phi}\mathbf{x} \tag{2}$$

Solving (2) directly is intractable as it is NP-hard. Consequently, many pieces of literature have been proposed to find an approximate solution to (2).

2.2. Greedy algorithms

Greedy algorithms iteratively approximate the original signal via residue minimization per iteration and employ an upper bound ε to limit the approximation error.

$$\hat{\mathbf{x}} = \operatorname{argmin} \|\mathbf{x}\|_0 \quad s.t. \quad \|\mathbf{y} - \mathbf{\Phi}\mathbf{x}\|_2 \le \varepsilon \tag{3}$$

As the basic one in greedy algorithms, OMP selects the dictionary atom which has the highest inner-product with the residue at each iteration, then introduces orthogonal projection of the residue onto the selected atoms. The second procedure enhances the reconstruction rate and speeds up the convergence. Because of simple implementation and low complexity, OMP has a significant impact on later OMP like algorithms.

2.3. Semi-greedy algorithm A*OMP

A*OMP transforms sparse signal reconstruction into a search for the correct index of vector $\mathbf{x}$ among dynamically evolving candidate subsets. These candidate subsets can be interpreted as different paths in a search tree, where each node represents a dictionary atom in $\mathbf{\Phi}$. The search tree starts with one or more roots, builds up and evaluates iteratively by A* search. The cost model for A* search to select the path can be formally described as

$$f(n) = g(n) + h(n) \tag{4}$$

where $g(n)$ represents the exact cost of the path from the starting point to any vertex n, and $h(n)$ represents the heuristic estimated cost from vertex n to the ending point. A* search balances the two as it moves from the starting point to the ending point. Each time through the main loop, it examines the vertex n that has the lowest $f(n)$ [24,25]. In the default model structure, A* search combines Dijkstra's algorithm $g(n)$ and Breadth-First-Search $h(n)$ that leads to being more intelligent than the other algorithms in graph theory.

As the most popular choice in pathfinding, A* search and its dynamic cost model have gained much attention. Particularly, the authors of [20] not only proposed a novel semi-greedy recovery approach named A*OMP, but also provided several practical cost models to realize it, including Additive Model, Adaptive Model and Multiplicative Model. For the convenience of the following analysis, we propose a united model for A*OMP, and it can be expressed as

$$\begin{cases} \left\|\hat{\mathbf{r}}_i^K\right\|_2 = \left\|\mathbf{r}_i^l\right\|_2 - \displaystyle\sum_{k=l+1}^{K} \hat{\delta}_i(k) \\ \hat{\delta}_i(k) = \left\|\hat{\mathbf{r}}_i^{k-1}\right\|_2 - \left\|\hat{\mathbf{r}}_i^k\right\|_2 \end{cases} \tag{5}$$

where $\|\mathbf{r}\|_2$ and $\|\hat{\mathbf{r}}\|_2$ denote the true residue and the estimated residue, respectively. $\hat{\delta}(k)$ denotes the estimated contribution to the decrease of the final residue. The superscript K and l denote the K-th and l-th iteration respectively, while the subscript i denotes the i-th path.

3. Nonlinear regression A*OMP

3.1. Problem definition and idea of NR-A*OMP

As described in previous sections, an improper cost model or inaccurate preset parameters may cause the degradation and deterioration of A*OMP. For instance, considering the l-th iteration in Adaptive Model, the cost model is given by