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DOA estimation based on the difference and sum coarray for coprime arrays

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ABSTRACT

In this paper, we construct a novel coarray named as the difference and sum (diff–sum) coarray by exploiting an improved Conjugate Augmented MUSIC (CAM) estimator, which utilizes both the temporal information and the spatial information. The diff–sum coarray is the union of the difference coarray and the sum coarray. When taking the coprime array as the array model, we find that the elements of the sum coarray can fill up all the holes in the difference coarray. Besides, the sum coarray contains bonus uniform linear array (ULA) segments which extend the consecutive range of the difference coarray. As a result, the consecutive lags of the diff–sum coarray are much more than those of the difference coarray. For analysis, we derive the hole locations and consecutive ranges of the difference set and the sum set, discuss the complementarity of the two sets, and provide the analytical expression of the diff–sum virtual aperture. Simulations verify the effectivity of the improved method and show the high DOF of the diff–sum coarray.

1. Introduction

Direction-of-arrival (DOA) estimation of multiple narrowband signals is a major application of the antenna array [1–7]. In the past few decades, many high-resolution subspace-based methods have been proposed for direction finding, such as MUSIC [2], ESPRIT [8] etc. By using these conventional methods, the number of detectable sources impinging on a uniform linear array (ULA) with N sensors is at most $N-1$ [9]. Detecting more sources than the number of sensors has been more and more attractive in recent years [10]. Constructing sparse arrays based on the concept of the coarray is a useful and important way to increase the degrees of freedom (DOF). In [11], the minimum redundancy array (MRA) was introduced for this purpose. It is a kind of sparse array whose difference coarray has no holes in it. However, there is no closed form expression for the MRA. As a result, the DOF for a given number of sensors can not be obtained as well.

To resolve these problems, several sparse arrays have been proposed recently. In [12], a nested array, which can detect at most $(N^2+2N)/4-1$ (N is even) or $(N+1)^2/4-1$ (N is odd) sources with only N sensors, was proposed. This structure is obtained by systematically nesting two ULAs. Another sparse array named the coprime array was proposed in [13]. Two uniform linear subarrays, which share the first sensor, are used to form the sparse array. One is M sensors with spacing N units, and the other is N sensors with spacing M units, where integers M and N are coprime. Coprime arrays can detect as many as $O(MN)$ sources using only $M+N-1$ sensors. However, compared to a nested array, a coprime array requires more sensors to achieve the same DOF. Thus, how to increase the DOF of the coprime array has generated a new wave of interest. An extended coprime array which contains $M+2N-1$ sensors was proposed in [14]. Its difference coarray lags can reach consecutive integers from $-MN-N+1$ to $MN+N-1$, which has been somewhat extended with more sensors used. Authors in [15] proposed the generalized coprime array concept with two operations. The first operation is compressing the inter-sensor spacing of one subarray in the coprime array, which yields a coprime array with compressed inter-element spacing (CACIS). The second operation is displacing one subarray in the coprime array, which forms a coprime array with displaced subarrays (CADiS). Both the CACIS and CADiS can achieve a high number of DOF. In addition to optimizing the design of the coprime array configuration, several novel methods have also been proposed to increase the DOF of the coprime array. In [16], a sparsity enforced recovery technique for the coprime array was proposed. Taking the off-grid DOAs into account, the method can construct a larger difference coarray than that in [14] by utilizing the grid offset vector. However, the closed form expression for the DOF has not been summarized. In [17], a novel coarray interpolation method for the coprime array was

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proposed. This method utilizes nuclear norm minimization to interpolate the missing samples or holes and is capable of achieving a high number of DOF. However, the actual freedom is still governed by the number of unique virtual sensors in the nonuniform difference coarray. Although all the geometries and methods mentioned above can correctly estimate more sources than the number of sensors, only the difference coarray is considered. The sum coarray cannot be utilized effectively.

Utilizing the difference and sum coarrays jointly to perform DOA estimation can further increase the DOF. The sum coarray usually arises as the virtual array in active sensing. With the transmitters illuminating the field of view and the receivers detecting the reflections from the targets, the received data can be considered as observations from the sum coarray. A sparsity-based method using active nonuniform arrays was proposed in [18] for extended aperture with both sum and difference coarrays. The vectorized covariance matrix of the sum coarray observations emulates the equivalent signals received from the difference coarray of the sum coarray. But one weakness of the method is that the sum coarray cannot perform sufficiently due to different transmitter and receiver arrays. In [19], a modified minimum redundancy monostatic multiple-input multiple-output (MIMO) configuration was proposed. As the transmitter and receiver arrays are identical, both the difference coarray and sum coarray can be used sufficiently. However, these methods constructing the sum coarray are restricted to active arrays. Therefore, proposing a novel method which can utilize the passive arrays to construct the difference and sum coarrays is significant.

In this paper, we exploit an improved Conjugate Augmented MUSIC (CAM) estimator [20], which we name as Vectorized Conjugate Augmented MUSIC (VCAM), to perform DOA estimation using passive arrays. By utilizing both the temporal information and the spatial information, we construct a conjugate augmented correlation vector based on the second-order statistics. Instead of calculating the fourth-order (FO) cumulants in [20], we vectorize the covariance matrix of the conjugate augmented correlation vector and get an equivalent received signal using the concept of Khatri–Rao (KR) product. The resulting coarray, which is named as the difference and sum (diff–sum) coarray in this paper, comprises not only the difference set but also the sum set. For the coprime array, it is at last verified that the elements of the sum coarray can fill up all the holes in the difference coarray. Besides, the sum coarray contains bonus ULA segments outside the consecutive range of the difference coarray. As a result, the virtual array can achieve much higher DOF than the difference coarray obtained by vectorized MUSIC [12]. Furthermore, its virtual aperture is more than twice the physical aperture, which will contribute to the decrease of the array size. In particular, we analyze the performance of the consisting sets of the diff–sum coarray and discuss the relationship between them. Based on the complementarity of the difference set and sum set, we give the expressions of the diff–sum coarray aperture for quantitative evaluation. Simulations verify the effectivity of the improved method.

The rest of the paper is organized as follows. In section 2, we first review the data model and the coprime array configuration. In section 3, we present the VCAM algorithm to acquire the diff–sum coarray. Then, in section 4, we derive the properties of the consisting difference coarray, the consisting sum coarray and the final diff–sum virtual array for the coprime geometry. Simulation results are provided in section 5 to numerically compare the performance of the nested array, the CACIS and the CADiS with vectorized MUSIC and the coprime array with VCAM method. Section 6 concludes the paper.

Notations. In this paper, vectors are denoted by italic boldface lowercase letters, e.g., $\boldsymbol{a}$. Matrices are denoted by italic boldface capital letters, e.g., $\boldsymbol{A}$. $(.)^H$ denotes conjugate transpose, whereas $(.)^T$ and $(.)^*$ respectively denote transpose and conjugation. $vec(.)$ is used to denote vectorizing operation and the symbol $\otimes$ denotes the left Kronecker product.

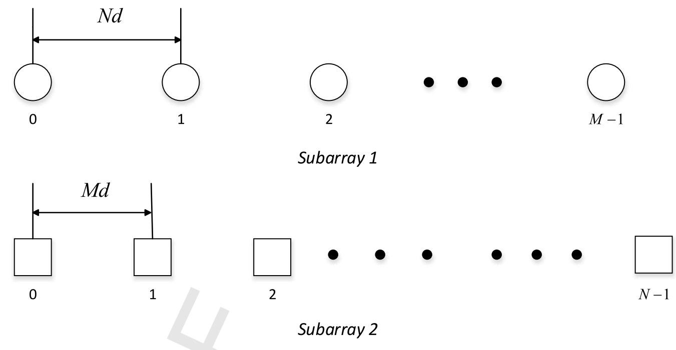

Fig. 1. The prototype coprime array.

2. System model

Denote $\boldsymbol{d} = \{d_1, \ldots, d_P\}$ as the positions of the array sensors with the first sensor as the reference, i.e., $d_1 = 0$. K uncorrelated narrowband plane wave sources impinge on the array from directions $\{\theta_1, \ldots, \theta_K\}$ with powers $\{\sigma_i^2, i = 1, 2, \ldots, K\}$. According to [20], we denote the ith source signal as $s_i(t) = u_i e^{j(\omega_c + \omega_i)t}$, where ω_c is the carrier frequency, u_i is the deterministic complex amplitude and ω_i is a small frequency offset. For different source signals, the frequency offsets are respectively different. After demodulation to IF, the ith signal becomes $s_i(t) = u_i e^{j\omega_i t}$. The additive noise is assumed to be white Gaussian with zero mean and variance σ_n^2, which is uncorrelated with the sources. Denote λ as the wavelength of the carrier. Then, the received signal can be expressed as

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k) s_k(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \ldots, \boldsymbol{a}(\theta_K)]$ with

$$\boldsymbol{a}(\theta_k) = [1, e^{j2\pi d_2 \sin(\theta_k)/\lambda}, \ldots, e^{j2\pi d_P \sin(\theta_k)/\lambda}]^T. \tag{2}$$

$\boldsymbol{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ and $\boldsymbol{n}(t) = [n_1(t), n_2(t), \ldots, n_P(t)]^T$ are source vector and noise vector, respectively. Since the element in the ith row and jth column of the covariance matrix

$$\boldsymbol{R}_{xx} = E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] \tag{3}$$

corresponds to the equivalent signal received from the sensor located at $d_i - d_j$, vectorizing $\boldsymbol{R}_{xx}$ can obtain the equivalent received signal at the difference coarray. It's noted that, in practice, the covariance matrix $\boldsymbol{R}_{xx}$ is estimated from a finite (say, N_p) snapshots as

$$\overline{\boldsymbol{R}}_{xx} = \frac{1}{N_p} \sum_{a=1}^{N_p} \boldsymbol{x}(a\Delta t)\boldsymbol{x}^H(a\Delta t),$$

where Δt is the sampling interval for snapshots. Δt is set to satisfy the sampling theory.

The coprime array, as shown in Fig. 1, is the union of two uniform linear subarrays [13]. One is of M sensors with the intersensor spacing of N units. The other is of N sensors with the inter-sensor spacing of M units. M and N are coprime integers. Assume that $M < N$. Then, the number of sensors in the coprime array is $P = M + N - 1$. Denote d as the unit inter-element spacing. The array sensors are located at

$$L_{prototype} = \{Nmd \,|0 \le m \le M-1\} \cup \{Mnd \,|0 \le n \le N-1\}. \tag{4}$$