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A low-complexity Lanczos-algorithm-based detector with soft-output for multiuser massive MIMO systems

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A B S T R A C T

Zero-Forcing (ZF) algorithm achieves the near-optimal detection performance for massive multiple-input multiple-output (MIMO) systems at expense of performing the complicated matrix inversion of a high dimensional matrix. In this paper, a novel Lanczos-algorithm-based signal detection method with soft-output is proposed to iteratively realize ZF algorithm for multiuser massive MIIMO systems, which avoids the exact computation of matrix inversion and in turn reduces the computational complexity from $O(K^3)$ to $O(K^2)$, where K denotes the number of users. In the development of the proposed method, by analyzing the iterative process of Lanczos algorithm, an approximate low-complexity scheme is proposed to calculate the log likelihood ratios (LLRs) for soft channel decoding. Simulation results show that the proposed detector provides a relatively good tradeoff between the complexity and performance compared with the several recently proposed detectors, and achieves almost the same performance as the ZF algorithm with only 3 iterations.

1. Introduction

Multiple-input multiple-output (MIMO) technology [1] can provide a notable improvement in data rate and reliability compared with single-input single-output (SISO) systems. In recent years, multiuser MIMO configurations, where the base stations (BSs) are equipped with multiple antennas and communicate with several cochannel users, have attracted much attention and built the foundation of several modern wireless communication standards, such as the 3GPP Long-Term Evolution Advanced (LTE-A) [2], IEEE 802.11n [3], to name a few. As an extension, the concept of massive MIMO [4] equipped with hundreds of antennas at BSs, compared to conventional (small-scale) MIMO systems, has been proposed to serve tens of users simultaneously and in the same frequency band. Theoretical results for massive MIMO not only promise huge advantages in terms of energy efficiency, spectral efficiency, and reliability compared to small-scale systems, but also simple and low-complexity signal detection algorithms achieve the optimal performance when the number of BS antennas approaches infinity [5]. Hence it is widely believed that the massive MIMO is an enabler for developments of future broadband (fixed and mobile) networks [6].

Unfortunately, the number of antenna configurations for the realistic massive MIMO systems is far from the infinity. As a consequence, it is the most critical task to design reliable and computationally efficient detectors in uplink massive MIMO systems. The well-known maximum likelihood (ML) detector provided optimal error performance, but it suffered from exponential complexity grow in terms of the number of transmit antennas. To address the stringent needs of massive MIMO detector, several non-linear signal detectors using the machine learning/artificial intelligence were proposed. The representative examples were the likelihood ascent search (LAS) detector [7] and reactive tabu search (RTS) detector [8]. However, the very close to ML performance for these two detectors has been reported only for low order modulation, such as BPSK and 4-QAM, and in fact their performances were far away from ML for higher-order QAM like 16-QAM and 64-QAM. To improve the performance for high-order QAM, multiple-output LAS detector [9] and random-restart RTS detector [10] were proposed, which involve considerably high complexity when the dimension of MIMO systems is large. Additionally, two near-ML detection methods, respectively termed randomized Markov chain Monte Carlo (R-MCMC) and randomized search (RS), were proposed in [11] for low order modulations. The near-ML performance in a 50×50 MIMO system was demonstrated using a Gibbs sampling based detection algorithm in [12], where the symbols take values from ± 1. In [13], a factor graph based belief propaga-

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tion (BP) algorithm was proposed for large-MIMO detection, where Gaussian approximation of the interference (GAI) is adopted.

The detectors mentioned above obtained closer to ML performance with the increased dimension of MIMO systems, but with higher complexity. Moreover, near-ML performance has been reported only when the ratio of number of BS antennas and antennas of the simultaneous users is close to one. According to [4], this will not be true for future massive multiuser MIMO systems, where the number of BS antennas is generally far more than that of the simultaneous single-antenna users, i.e., low system loading factors. For the multiuser massive MIMO systems with low system loading factors, the random channel vectors between the users and the BS become pairwisely orthogonal when the number of BS antennas grows large [14]. This merit favors the linear detectors based on zero-forcing (ZF) and minimum mean square error (MMSE) criterion with near-ML performance for uplink multiuser massive MIMO systems [4]. However, these two detectors involve unfavorable matrix inversion in which complexity is proportional to the cubed number of antennas of the simultaneous users. In order to avoid computing the exact matrix inversion, Wu M. et al. proposed an approximate matrix inversion method relying on a Neumann series expansion [15], which still had complexity of $O(K^3)$ when approximate order was equal to or greater than 3. Subsequently, the conjugate gradient (CG) method was used for data detection and precoding in order to improve upon the Neumann series approach for massive MIMO systems [16], while the Gauss–Seidel (GS) method was exploited to iteratively realize the MMSE algorithm without the direct matrix inversion [17]. In [18], an iterative signal detector based on symmetric successive over-relaxation (SSOR) method was proposed to reduce the complexity of ZF detector by about one order of magnitude. Based on MMSE criterion with the symmetric complex bi-conjugate gradients (SCBICG) and Lanczos algorithm, two soft-output detectors were proposed in [19] in which the detector using the Lanczos algorithm not only stored many intermediate results, but also involved explicitly computing the inversion of symmetric tridiagonal matrix in order to obtain the soft information. It led to the detector using the Lanczos algorithm the dramatic increase of computation and storage. In [20], a Lanczos-based signal detector based on the hard-decision method was proposed for massive MIMO systems, where the Lanczos algorithm realizes the tri-diagonalization process and deduces the iteration equation of the transmitted signal vector to reduce the computation complexity. Based on joint steepest descent algorithm and Jacobi iteration, an iterative linear detector was proposed for uplink massive MIMO systems in [21]. More recently, an approximate iterative linear MMSE detector was developed in [22].

In this paper, according to the ZF criterion, a soft-output detector based on Lanczos algorithm [23,24] is presented for uplink massive multiuser MIMO system with near-optimal performance. The detector firstly exploits the special structure of the ZF filtering matrix to iteratively perform the signal detection based on Lanczos algorithm without matrix inversion. In the method, to compute the soft-output information, i.e., log likelihood ratios (LLRs), by analyzing the iterative procedure of Lanczos algorithm, the expressions of the true and approximate computation of the LLRs are derived theoretically. Simulation results verify that the approximated method for computing the LLRs obtain the near-optimal performance with much lower complexity. Additionally, the proposed method provides the recursive relationship of the intermediate variables between consequent two iterations, which leads to a dramatic decrease of storage load.

This paper is organized as follows: In Section 2, the system model is presented. In Section 3, the soft-output detector based on Lanczos algorithm is proposed, and its complexity is analyzed. In Section 4, simulation results are presented. Finally, Section 5 concludes the paper.

2. System model

Consider the uplink of a multiuser massive MIMO system with K independent users, where each user is equipped with one transmit antenna, and the BS is equipped with an array of N antennas ($K \ll N$). The K users encode their own information bits and map into M-QAM constellation points in the finite alphabet Ω with cardinality $|\Omega|$ and average transmit power is E_s per symbol. Let $\mathbf{x}_c$ represent the $K \times 1$ transmitted signal vector consisting of the transmitted symbols from K users, the $N \times 1$ received signal vector $\mathbf{y}_c$ at the BS is

$$\mathbf{y}_c = \mathbf{H}_c \cdot \mathbf{x}_c + \mathbf{n}_c, \tag{1}$$

where $\mathbf{n}_c$ is the noise vector whose entries are modeled as i.i.d. $\mathbb{CN}(0, \sigma^2)$ and $\mathbf{H}_c$ denotes the $N \times K$ complex channel gain matrix. According to [4] and [5], the entries of $\mathbf{H}_c$ can further be modeled as i.i.d random variables with zero mean and unit variance for $K \ll N$. In what follows, $\mathbf{H}_c$ is assumed to be perfectly estimated at BS and the signal-to-noise ratio (SNR) is defined by $K \cdot E_s/\sigma^2$.

Let $\mathbf{y} = [\mathrm{Re}\{\mathbf{y}_c\}\ \mathrm{Im}\{\mathbf{y}_c\}]^{\mathrm{T}}$, $\mathbf{x} = [\mathrm{Re}\{\mathbf{x}_c\}\ \mathrm{Im}\{\mathbf{x}_c\}]^{\mathrm{T}}$, and $\mathbf{n} = [\mathrm{Re}\{\mathbf{n}_c\}\ \mathrm{Im}\{\mathbf{n}_c\}]^{\mathrm{T}}$, we rewrite the complex system model in (1) into a real-value one as

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{n}, \tag{2}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathrm{Re}\{\mathbf{H}_c\} & -\mathrm{Im}\{\mathbf{H}_c\} \\ \mathrm{Im}\{\mathbf{H}_c\} & \mathrm{Re}\{\mathbf{H}_c\} \end{bmatrix}$$

with $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ representing the real and complex parts of a complex vector or matrix, respectively.

The typical ZF detection is considered for (2), which can be represented by

$$\widehat{\mathbf{x}} = \mathbf{F} \cdot \mathbf{y} = \mathbf{x} + \bar{\mathbf{n}}, \tag{3}$$

where $\widehat{\mathbf{x}}$, $\mathbf{F} = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}$ and $\bar{\mathbf{n}} = \mathbf{F} \cdot \mathbf{n}$ are the corresponding estimated vector, the ZF equalization matrix in real form, and the post-equalization noise vector, respectively.

After the ZF detection, LLRs can be extracted for channel decoder. Let $\mathbf{J} = \mathbf{H}^{\mathrm{T}}\mathbf{H}$ and $\bar{\mathbf{y}} = \mathbf{H}^{\mathrm{T}}\mathbf{y}$, we have $\widehat{\mathbf{x}} = \mathbf{J}^{-1}\bar{\mathbf{y}}$. Combining (1) and (2), expression $\widehat{\mathbf{x}} = \mathbf{x} + \bar{\mathbf{n}}$ can be rewritten into complex form,

$$\widehat{\mathbf{x}}_c = \mathbf{x}_c + \bar{\mathbf{n}}_c, \tag{4}$$

where $\bar{\mathbf{n}}_c = \mathbf{F}_c \cdot \mathbf{n}_c$ with $\mathbf{F}_c = ({\mathbf{H}_c}^{\mathrm{H}}\mathbf{H}_c)^{-1}{\mathbf{H}_c}^{\mathrm{H}}$ representing the ZF equation matrix in complex form. Let $\widehat{x}_{c,i}$, $x_{c,i}$ and $\bar{n}_{c,i}$ represent ith element of the $\widehat{\mathbf{x}}_c$, $\mathbf{x}_c$ and $\bar{\mathbf{n}}_c$, respectively, and based on a Gaussian approximation for (4), we have

$$\widehat{x}_{c,i} \sim \mathbb{CN}(x_{c,i}, v_{c,i}^2) \tag{5}$$

where variance $v_{c,i}^2 = \sigma^2 \cdot \|\mathbf{F}_{c,i}\|_2^2$ with $\mathbf{F}_{c,i}$ denoting the ith row of matrix $\mathbf{F}_c$. On the basis of normalizing the transmit power $E_s = 1$, the max-log approximated LLR of the bth bit $x_{c,i,b}$ in modulated symbol $x_{c,i}$ for user i can be computed as follows

$$L(x_{c,i,b}) = \frac{1}{v_{c,i}^2}\left(\min_{s\in\Omega_b^0} \left|\widehat{x}_{c,i} - s\right|^2 - \min_{s\in\Omega_b^1} \left|\widehat{x}_{c,i} - s\right|^2\right), \tag{6}$$

where $i = 1, 2, \cdots, K$, $b = 1, 2, \cdots, \log_2{}^M$, and the sets Ω_b^0 and Ω_b^1 contain the constellation symbols where the ith bit of the symbols in Ω equals 0 and 1, respectively. In fact, $1/v_{c,i}^2$ in (6) is the post-equalization SNR (P-SNR).