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Matrix CFAR detectors based on symmetrized Kullback–Leibler and total Kullback–Leibler divergences

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ABSTRACT

Target detection in clutter is a fundamental problem in radar signal processing. When the received radar signal contains only few pulses, it is difficult to achieve a satisfactory performance using the traditional detection algorithm. In recent times, a generalized constant false alarm rate (CFAR) detector on the Riemannian manifold of Hermitian positive-definite (HPD) matrix was proposed. The employment of this detector, which compares the Riemannian distance between the covariance matrix of the cell under test (CUT) and an average matrix of reference cells with a given threshold, has significantly improved the detection performance. However, the application of this detector in real scenarios is still limited by two problems; it is computationally expensive and the detection performance is not very good since the Riemannian distance is utilized. In this paper, the symmetrized Kullback–Leibler (sKL) and the total Kullback–Leibler (tKL) divergences, instead of the Riemannian distance, are used as dissimilarity measures in the matrix CFAR detector. According to sKL and tKL divergences, three average matrices, the sKL mean, the sKL median, and the tKL t center, are derived. Furthermore, the relationship between the detection performance and the anisotropy of the distance measure used in the matrix CFAR detector is explored. Numerical experiments and real radar sea clutter data are given to confirm the superiority of the proposed algorithms in terms of the computational complexity and the detection performance.

1. Introduction

Target detection in a clutter is of great importance in radar signal processing when only few pulses are available. However, the classical fast Fourier transform (FFT) based constant false alarm rate (CFAR) detection algorithms [1] cannot achieve satisfactory performance owing to the poor Doppler resolution as well as the energy spread of the Doppler filter banks. To circumvent these drawbacks, many strategies are conceived to cope with such situations. For instance, in [2–6], the authors exploit the priori information about the surrounding environment to achieve significant performance improvements. Another example is provided in [7], wherein the Bayesian approach is employed to assume a suitable distribution about the unknown clutter covariance matrix, and similar methods are found in [8–10].

In recent times, a generalized CFAR technique on the Riemannian manifold of the Hermitian positive-definite (HPD) matrix, referred to as the Riemannian distance-based matrix CFAR, was proposed by F. Barbaresco [11–13]. In this algorithm, the received radar complex signal $\boldsymbol{z}$ ($\boldsymbol{z} = \{z_0, z_1, \ldots, z_{n-1}\}$ represents the set of n pulses) in each cell in one coherent processing interval (CPI) is modeled as a complex circular multivariate Gaussian distribution with zero mean, which can be represented by its covariance matrix $\boldsymbol{R} = \mathbb{E}[\boldsymbol{z}\boldsymbol{z}^H]$. The detection procedure of the Riemannian distance-based matrix CFAR detector can be formulated as follows: For each cell under test (CUT), the Riemannian distance between the covariance matrix of CUT and the Riemannian mean or median matrix of reference cells around the CUT is computed; if this distance is greater than a given threshold determined by the Monte Carlo method in order to maintain the false alarm constant, then one can conclude that there is a target at the location of the CUT. It has been proved that the Riemannian distance-based matrix CFAR detector has better detection performance than the classical FFT-CFAR detection algorithm [12,13]. However, there are two shortcomings in the Riemannian distance-based matrix CFAR detector: the computational cost is very high; the detection performance is not very good.

To meliorate the above deficiencies, a straight conception is to explore some alternative distance measures on the Riemannian manifold instead of the Riemannian distance. Simultaneously, these distance measures have lower computational complexity and better detection performance than the Riemannian distance used in the matrix CFAR detector. In addition to the Riemannian distance,

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many of the measure structures can be given on the Riemannian manifold of HPD matrices. For instance, the square loss function has been applied to regression analysis; the Bhattacharyya divergence is exploited for diffusion tensor magnetic resonance image (DT-MRI) segmentation [14,15]; and the Kullback–Leibler (KL) divergence [16] has been used for measuring the dissimilarity between two probability density functions. The KL divergence is an information measure already widely used in many detection problems, even involving radar signals. In the multi-target recognition application, the mutual information and the KL divergence are used for the multiple-input multiple-output (MIMO) radar optimal waveform design [17]. In [18], the authors employ the KL divergence to measure the difference between the probability densities of the observations under two alternative hypotheses. Moreover, the difference is applied to address the problem of space–time code design for a MIMO radar detection system. In the synthetic aperture radar (SAR) image change detection application, the problem of change detection on SAR images acquired at different dates, is addressed via the non-parametric estimation [19] and the parametric estimation [20,21] of the KL divergence is used as a distance between statistical distributions of local features. Moreover, the KL divergence has been applied to anomaly detection with the wall human detection [22] and the monitoring of large-scale technical systems [23], Stereo matching [24], incipient fault detection [25], and the optimal distribution approximation [26]. In radar target detection applications, our previous work [27] and Zhao [28] have explored the matrix CFAR detection method based on the KL divergence. Numerical experiments [27,28] and X-band radar clutter [12] are given to confirm that the detection performance of KL divergence-based matrix CFAR detector outperforms the Riemannian distance-based matrix CFAR detector, and both these two matrix CFAR detection algorithms have better detection performance than the classical FFF-CFAR.

Among many modern radar applications, a class of detector is based on the Cell-Averaging (CA) CFAR technique. The cell-averaging process is an example of what is known as a sliding window detector. It is based upon the properties of the Gaussian processes instead of the Neyman–Pearson Lemma. The matrix CFAR detector [12,27] and the classical CA-CFAR detector [1] are of similar schemes under the constant false alarm rate formulation. The slow-time dimension of the received clutter data in each cell is modeled and represented as a HPD matrix. This HPD matrix denotes the correlation or power level between multiple pulses, and is estimated by the pulses data according to its correlation coefficient. The matrix CFAR detector differs from the CA-CFAR detector in three ways: (1) the observation data in each cell is a HPD matrix, and not the original pulse data; (2) the distance measures used in matrix CFAR detector is Riemannian distance or divergence measure, and not the Euclidean distance; and (3) the averaging process in matrix CFAR detector is geometric mean of a set of HPD matrices, not the arithmetic mean of scalar number. These differences imply that the matrix CFAR detector performs on the HPD matrix space, in other words, the different geometry considered in detection.

In this paper, we develop the detection algorithms in the framework of the matrix CFAR detector based on the symmetrized Kullback–Leibler (sKL) divergence [29] and the total Kullback–Leibler (tKL) divergence [30]. According to sKL and tKL divergences, three average matrices, the sKL mean [31], median, and the tKL t center [30], are derived and explored to replace the Riemannian mean and median. Furthermore, we explore the relationship between the detection performance of the matrix CFAR detectors based on different distance measures and the anisotropy of the distance measure. We show that different distance measures have different anisotropy, and the distance measure, which has a better anisotropy, has a better detection performance. These results would have been proven by numerical experiments and real sea clutter data.

The rest of this paper is organized as follows: Section 2 gives a brief description of the Riemannian distance-based matrix CFAR detector; the detector proposed in this paper is detailed in Section 3; results obtained from simulated data are presented in Section 4; and Section 5 concludes our work.

1.1. Notation

A lot of notations are adopted as follows. We use math italic for scalars x, uppercase bold for matrices $\boldsymbol{A}$, and lowercase bold for vectors $\boldsymbol{x}$. The conjugate transpose operator is denoted by the symbol $(\cdot)^H$. $tr(\cdot)$ and $\det(\cdot)$ are the trace and the determinant of the square matrix argument, respectively. $\boldsymbol{I}$ denotes the identity matrix, and $\mathbb{C}^n$, $\mathbb{H}(n)$ are the sets of n-dimensional vectors of complex numbers and of $n \times n$ Hermitian matrices, respectively. The Frobenius norm of the matrix $\boldsymbol{A}$ is denoted by $\|\boldsymbol{A}\|_F$. For any $\boldsymbol{A} \in \mathbb{H}(n)$, $\boldsymbol{A} > 0$ means that $\boldsymbol{A}$ is a HPD matrix, and denoted by $\mathbb{P}(n)$. Finally, $\mathbb{E}(\cdot)$ denotes statistical expectation.

2. Riemannian distance-based matrix CFAR detector

2.1. Signal model and signal manifold

For the radar received complex clutter data $\boldsymbol{z} = \{z_0, z_1, \ldots, z_{n-1}\}$ in each cell in one CPI, where n is the length of pulses, assuming $\boldsymbol{z}$ is a complex circular multivariate Gaussian distribution, $\boldsymbol{z} \sim CN(\boldsymbol{0}, \boldsymbol{R})$, with zero mean and covariance matrix $\boldsymbol{R}$,

$$\boldsymbol{p}(\boldsymbol{z}|\boldsymbol{R}) = \frac{1}{\pi^n \det(\boldsymbol{R})} \exp\left\{-\boldsymbol{z}^H \boldsymbol{R}^{-1} \boldsymbol{z}\right\} \tag{1}$$

with the covariance matrix $\boldsymbol{R}$ given by

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{z}\boldsymbol{z}^H\right] = \begin{bmatrix} r_0 & \bar{r}_1 & \cdots & \bar{r}_{n-1} \\ r_1 & r_0 & \cdots & \bar{r}_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ r_{n-1} & \cdots & r_1 & r_0 \end{bmatrix},$$
$$r_k = \mathbb{E}[z_i \bar{z}_{i+k}], \quad 0 \le k \le n-1, 0 \le i \le n-1 \tag{2}$$

where $r_k = \mathbb{E}[z_n \bar{z}_{n+k}]$ is called the correlation coefficient and $\bar{z}$ denotes the complex conjugate of z. $\boldsymbol{R}$ is a Toeplitz HPD matrix with $\boldsymbol{R}^H = \boldsymbol{R}$. The estimation of covariance matrix $\boldsymbol{R}$ is called the direct data domain method, which has been used in array signal processing [32] and space time adaptive processing [33]. It is well known that the stationary Gaussian processes have both ergocity and strict stationarity. According to the ergodicity, the correlation coefficient r_k of data $\boldsymbol{z}$ can be calculated by averaging it over time instead of its statistical expectation $\mathbb{E}[z_n \bar{z}_{n+k}]$, as

$$\hat{r}_k = \frac{1}{n-k} \sum_{j=0}^{n-1-k} z_j \bar{z}_{j+k}, \quad 0 \le k \le n-1 \tag{3}$$

The pulse data in each cell in one CPI is modeled by Eqs. (1) and (2), and are represented by an HPD matrix. This matrix stands for the correlation or power level between multiple pulses. Through parameterization by the HPD matrix, the received clutter data in each cell in one CPI $\boldsymbol{z} = \{z_1, z_2, \ldots, z_n\}$ can be mapped into an n dimensional parameter space,

$$\boldsymbol{\psi}: \mathbb{C}^n \to \mathbb{P}(n), \quad \boldsymbol{z} \to \boldsymbol{A} \in \mathbb{P}(n) \tag{4}$$

Here, $\mathbb{P}(n)$ forms a differentiable Riemannian manifold [34]. In the following, we present the distance measure and its mean and, median on manifold.