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Fast algorithms for sparse frequency waveform design with sidelobe constraint

Liang Tang *, Yongfeng Zhu, Hao Wu, Qiang Fu

ATR key laboratory, National University of Defense Technology, Changsha, Hunan 410073, China

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ABSTRACT

Sparse frequency waveform (SFW) is widely used in wideband systems (for instance, radar and communication systems) to suppress the narrowband interferences or avoid the reserved frequency bands. However, it suffers high autocorrelation sidelobes due to the discontinuous spectrum. In this paper, we aim at designing waveforms with low autocorrelation sidelobes and arbitrary frequency stopbands. After formulating this design as an unconstrained minimization problem, two algorithms based on the majorization–minimization method and a gradient-based algorithm are derived to deal with this problem. The proposed algorithms can be easily implemented by the fast Fourier transform (FFT) operations and thus are computationally efficient for long waveform design with good sidelobe and stopband suppression. In addition, they can handle both the design problem of low sidelobe waveform and the design problem of sparse frequency waveform with low sidelobes. Numerical experiments show that the proposed algorithms can provide better performance than the existing ones on the running time.

1. Introduction

With the rapid development of the electronic technology, the competition of spectrum resources is becoming more and more fierce, thus the radar and communication systems have to work in the same congested frequency bands [1]. The result of spectral cohabitation is that the systems which share the frequency band are confronted with the narrowband interference problem [2]. An efficient way to overcome this problem is to design the sparse frequency waveform (SFW) which can improve the quality of wireless service and the detection capability of radar.

SFW is a kind of waveform with several discontinuous frequency stopbands. It has a wide range applications in ultra-wide bandwidth (UWB) systems [3,4], high frequency surface wave radar (HFSWR) [5–8] and cognitive radar [9]. In 2004, [10] applied the steepest descent (SD) method to design the sparse frequency transmit waveform, and proposed a mismatch filter to suppress the range sidelobes. Although [10] can achieve arbitrary frequency suppression, the transmit-receive waveform design method was time consuming, and would cause the loss of signal to noise ratio (SNR). To avoid the loss of SNR, the cyclic iteration [11] and the Lagrange programming neural network (LPNN) [12] were proposed to design waveforms with desired spectrum shapes. However, without considering the sidelobe constraint, the designed waveforms suffer the high range sidelobes, which is not desired in many applications [13–15].

In addition to the sparse frequency property, low autocorrelation sidelobes is also a good property of the waveform [16]. In recent years, the design of waveform with low autocorrelation sidelobes has attracted lots of attentions [17–24]. The main research can be divided into three categories: the design of low integrated sidelobe level (ISL) waveform [17–20], the design of low weighted ISL (WISL) waveform [19,21–23] and the design of low peak sidelobe level (PSL) waveform [21,24]. To design waveform with good correlation property, many efficient algorithms based on the fast Fourier transform (FFT) operations were developed, for instance the CAN (cyclic algorithm new) [19], WMISL-Diag-acc (monotonic minimizer for weighted ISL) [21] and DPM (discrete phase method) [24].

In order to suppress both the frequency stopbands and the sidelobes, many scholars were devoted to design SFW with low autocorrelation sidelobes [6,7,25,26]. Actually, the design of such waveform can be regarded as a bi-objective optimization (also known as Pareto optimization) problem due to the simultaneous consideration of two constraints. To solve the bi-objective problem, the scalarization procedure was applied to obtain a single-objective optimization problem which can be easily handled by many optimization methods. At the early stage, the genetic algorithm (GA) [6] and particle swarm optimization (PSO) algorithm

* Corresponding author.
E-mail addresses: tangliangcool@163.com (L. Tang), zoyofo@163.com (Y. Zhu), wowhow@163.com (H. Wu), fuqiang1962@vip.sina.com (Q. Fu).

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[7] were applied to tackle this problem. However, the convergence speeds of these algorithms are slow. To improve the computational efficiency, the cyclic algorithms named SCAN (stopband cyclic algorithm new) and WeSCAN (weighted SCAN) were proposed in [25]. Compared to the SCAN, the WeSCAN required more computations but is more flexible in waveform design. Meanwhile, [26] proposed a new cyclic algorithm (which we call SD-based cyclic algorithm, i.e., SDCA) that incorporates the SD method. However, the shortcoming of the SDCA is that the computation of the iteration step along the gradient direction is still complicated and costly. In addition, there are some other aspects of research on the sparse frequency waveform design [27–30]. In [27–29], the trade-off among the signal to interference plus noise ratio (SINR), spectral shape and the autocorrelation sidelobes of the waveform was studied, while [30] carried out the sparse frequency waveform analysis and design by using the ambiguity function theory.

Although there are some literatures on the SFW design, the computational complexity of the existing algorithms are so intolerable that the waveform online design in the dynamic electromagnetic environment is difficult to achieve. In this paper, we deal with the problem of designing SFW with low autocorrelation sidelobes. According to the WISL metric and the power spectrum density (PSD), a new objective function is established in the frequency domain via the scalarization procedure. By applying the majorization–minimization (MM) method three times, the majorization-based iterative algorithm (MIA) is developed. Due to the nature of the majorization function, the convergence speed of the MIA is slow. Thus, an acceleration scheme is applied to accelerate the MIA, and then we obtain the resulting accelerated MIA (AMIA). The majorization process analysis is then carried out. To further improve the convergence speed, the gradient-based iterative algorithm (GIA) is derived by minimizing the objective function directly. In GIA, the searching step size which is hard to calculate directly is deduced via the Taylor series expansion. Different from the existing algorithms, both the AMIA and the GIA can be implemented by the fast Fourier transform (FFT) operations, and have high computational efficiency.

The rest of the paper is organized as follows. In Section 2, the problem of designing SFW with sidelobe constraint is formulated. In Section 3, MIA and AMIA are developed based on the MM method, followed by the majorization process analysis. In Section 4, we develop the GIA by deducing the phase gradient and the step size, and then give a brief analysis of complexity. Section 5 provides some numerical experiments to verify the effectiveness of the proposed algorithms. Finally, Section 6 gives the conclusions.

Notation: Boldface upper case letters denote matrices while boldface lower case letters denote column vectors. The complex conjugate, transpose and conjugate transpose are denoted by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, respectively. $\|\cdot\|$ and $\|\cdot\|_F$ denote the Euclidean norm of the vector and the Frobenius norm of the matrix. $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote the real and imaginary part respectively. $\mathrm{Diag}(\mathbf{x})$ denotes a diagonal matrix formed with the column vector $\mathbf{x}$. $\circ$ denotes the Hadamard product. $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix. $\mathbf{x}(m)$ denotes the m-th element of the vector $\mathbf{x}$. $\mathbf{x}^k$ is the k-th iteration of $\mathbf{x}$. $\mathbf{X}_{:,1:i}$ denotes the submatrix formed with the first i columns of $\mathbf{X}$. $\mathbf{x}_{1:i}$ denotes the first i elements of $\mathbf{x}$. $\mathbf{I}$ denotes the identity matrix. $\mathbf{0}_N$ is the all-zero vector of length N. $\mathcal{F}(\mathbf{x})$ and $\mathcal{F}^{-1}(\mathbf{x})$ denote the $2N$-point FFT and IFFT (inverse FFT) operations of $\mathbf{x}$ respectively. In the (I)FFT operations, if the length of $\mathbf{x}$ is less than $2N$, $\mathbf{x}$ is padded with trailing zeros to length $2N$.

2. Problem formulation

As mentioned in introduction, this paper focuses on the problem of designing sparse frequency waveform with sidelobe constraint. Therefore, the waveform should satisfy two constraints: one is the sparse frequency constraint; and the other one is the sidelobe constraint. In the following, we first establish the corresponding criterions of these two constraints, and then formulate the waveform design problem.

2.1. The sparse frequency constraint

Let $\{x_n\}_{n=1}^N$ be the complex waveform to be designed. The vector form of the waveform can be expressed as

$$\mathbf{x} = [x_1, x_2, \ldots, x_N]^T. \tag{1}$$

Assume $\mathbf{f}$ is the $2N$-point discrete Fourier transform (DFT) vector of the waveform $\{x_n\}_{n=1}^N$, i.e.,

$$\mathbf{f}(m) = \sum_{n=1}^{N} x_n e^{-jn\omega_m}, \omega_m = \frac{2\pi}{2N} m, m = 1, \ldots, 2N, \tag{2}$$

then the power spectrum density (PSD) vector $\mathbf{p}$ of $\mathbf{x}$ can be written as

$$\mathbf{p} = \mathbf{f} \circ \mathbf{f}^*. \tag{3}$$

Waveform with sparse frequency property means that the PSD of the waveform has several frequency stopbands. Without loss of generality, we consider the frequency is normalized. Define the set of frequency stopbands as

$$\Omega_f = \bigcup_{k=1}^{n_s} (f_{k1}, f_{k2}) \subset [0, 1], \tag{4}$$

where (f_{k1}, f_{k2}) denotes one stopband, and n_s denotes the number of the stopbands. Since the frequency of the PSD corresponds to the normalized frequency $[0, 1]$, the suppression of the stopband Ω_f is equivalent to minimizing the PSD vector $\mathbf{p}$ in following intervals

$$\bar{\Omega}_f = \bigcup_{k=1}^{n_s} (2Nf_{k1}, 2Nf_{k2}) \subset [0, 2N]. \tag{5}$$

For example, assume that the stopband and waveform length are $(0.2, 0.3)$ and 256, then the PSD which should be minimized is $\mathbf{p}(k), k \in [103, 153]$. Define the frequency weight vector as

$$\mathbf{w}_f = [\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_{2N}]^T, \quad \bar{w}_p = \begin{cases} 1, & p \in \bar{\Omega}_f \\ 0, & \text{otherwise} \end{cases}. \tag{6}$$

Similar to the definition of the weighted integrated sidelobe level (WISL) [19,21], then the criterion of sparse frequency constraint can be formulated as

$$O_{PSD} = \sum_{k=1}^{2N} \bar{w}_k |\mathbf{p}(k)|^2 = \mathbf{p}^H \mathrm{Diag}(\mathbf{w}_f)\mathbf{p}. \tag{7}$$

Thus, by minimizing (7), the spectral power in Ω_f can be suppressed.

2.2. The sidelobe constraint

The commonly used criterion of the sidelobe suppression is the integrated sidelobe level (ISL) [17,19]:

$$\mathrm{ISL} = \sum_{k=1}^{N-1} |r_k|^2, \tag{8}$$

where r_k denotes the aperiodic autocorrelation function (ACF) of the waveform which is defined as