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11 A novel approach for time frequency localization of scaling functions 17 $\frac{11}{12}$ A novel approach for time–frequency localization of scaling functions $\frac{77}{78}$ ¹³ and design of three-band biorthogonal linear phase wavelet filter **the contact of the contact of the contact** 14 **1 1** 15 Daling 81 banks

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22 ARTICLE INFO ABSTRACT 88 23 89

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²⁴ Article history: **Example 20** 24 article history: **Design of time-frequency localized filters and functions is a classical subject in the field of signal** ²⁵ Available online xxxx **the state of the constant of the constant** principle states that a function cannot be localized in time and frequency ⁹¹ $\frac{26}{100}$ $\frac{25}{100}$ $\frac{25}{100}$ domain simultaneously and there exists a nonzero lower bound of 0.25 on the product of its time 27 *intywards* carrier called time–frequency variance and frequency variance called time–frequency product (TFP). Using arithmetic mean (AM)– 93 28 Throe reduction that the second through the second through the second of variances and sum of variances can be related and it can be 94 29 DFT 20 DET Shown that sum of variances has lower bound of one. In this paper, we compute the frequency variance of 30 96 the filter from its discrete Fourier transform (DFT) and propose an equivalent summation based discrete-₃₁ Prolate spheroid sequence time uncertainty principle which has the lower bound of one. We evaluate the performance of the $\,$ ₉₇ ₃₂ 92 broposed discrete-time time–frequency uncertainty measure in multiresolution setting and show that the 33
and frequency domain than that obtained from the Slepian, Ishii and Furukawa's concentration measures. ¹⁰⁰ 100
The proposed design approach provides the flexibility in which the TFP can be made arbitrarily close ¹⁰¹ 101 to the lowest possible lower bound of 0.25 by increasing the length of the filter. In the other proposed ¹⁰² 102 approach, the sum of the time variance and frequency variance is used to formulate a positive definite ³⁷ 37 103 matrix to measure the time–frequency joint localization of a bandlimited function from its samples. We 38 104 design the time–frequency localized bandlimited low pass scaling and band pass wavelet functions using 39 105 the eigenvectors of the formulated positive definite matrix. The samples of the time–frequency localized ⁴⁰ 106 bandlimited function is obtained from the eigenvector of the positive definite matrix corresponding to $_{41}$ and the state of the Tep of the designed bandlimited scaling and wavelet functions are close $_{107}$ 42 to the lowest possible lower bound of 0.25 and 2.25 respectively. We propose a design method for 108 43 109 time–frequency localized three-band biorthogonal linear phase (BOLP) wavelet perfect reconstruction 44 110 the synthesis basis functions for the specified frequency variance of the analysis scaling function. The ⁴⁵ 111 performance of the designed filter bank is evaluated in classification of seizure and seizure-free EEG ¹¹² 112 signals. It is found that the proposed filter bank outperforms other existing methods for the classification 47 113 of seizure and seizure-free EEG signals. proposed DFT based concentration measure generate sequences which are even more localized in time filter bank (PRFB) wherein the free parameters can be optimized for time–frequency localization of

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1. Introduction

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53 **1. Introduction 1988 and 1997 in the set of the set** 54 120 [\[4–6\],](#page--1-0) image coding [\[7–9,8,10\],](#page--1-0) edge detection and image segmen-55 In the last two decades, wavelet filter banks have played a key tation [11,12]. The joint time–frequency localization of the wavelet 121 56 vole in describing nonstationary signals such as speech, seismic, basis functions is generally represented by the dimensions of the lead 57 123 radar, electroencephalogram (EEG), electrocardiogram (ECG) [\[1–3\].](#page--1-0) 58 The basis functions generated by the wavelet filter bank provide area of the tile or the time–frequency product (TFP) represents the 124 59 125 multiple resolutions in time domain and frequency domain. Time– 60 126 domain. Various signal processing applications require trade-off in 61 127 time localization and frequency localization. For example, time lo-62 E -mail addresses: bhatidinesh@gmail.com (D. Bhati), pachori@iiti.ac.in **Calized bases preserve the edges in the image whereas bandwidth** 128 63 129 compression of an image can be achieved with good frequency lotation [\[11,12\].](#page--1-0) The joint time–frequency localization of the wavelet basis functions is generally represented by the dimensions of the tile on the time–frequency plane $[13]$ that represents them. The area of the tile or the time–frequency product (TFP) represents the joint localization of the basis function in the time and frequency

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17 certainty principle (HUP) for signals in $L_2(\mathbb{R})$ and the later is based In this paper, the notions of prolate spheroid wave functions 83 time–frequency localization of discrete scaling filters as well as for time-limited functions proposed by Bhati et al. [\[16\]](#page--1-0) and propose a design method for the time–frequency localized three-band and regularity order of one. The design problem of time–frequency parameters of the filter bank.

²⁶ There are various notions in the literature to measure the ef-**Changle 12** larity. In this work, we use a scaling filter with regularity order of $\frac{92}{1000}$ ²⁷ fective support of a function in time or frequency domain [\[24,14,](#page--1-0) ϵ two and design a time–frequency localized scaling function with ϵ ⁹³ 28 [5,15,25\].](#page--1-0) Slepian et al. [\[14\]](#page--1-0) measures the energy concentration in Sobolev regularity of two. HUP for linear operators is used in the 94 ²⁹ the given frequency band for time-limited signals whereas, Gabor second approach subject to the linear constraints for regularity ⁹⁵ 30 uses the notion of variances to measure the effective supports in to design time-frequency localized scaling filter and the function. 96 ³¹ time and frequency domains [\[5\].](#page--1-0) Time–frequency uncertainty prin-
Sharma et al. [31] use the notion of quantum harmonic oscillator $\frac{97}{100}$ 32 ciple (UP) in discrete domain is studied by several authors in the and DFT and propose the discrete-time UP which is an equivalent 38 ³³ literature. Ishii and Furukawa [\[15\]](#page--1-0) proposed time–frequency UP for of Gabor's UP for continuous time functions. In this paper, we com- 99 34 discrete time sequences. Gabor showed that signals in $L_2(\Re)$ can-
pute the frequency variance of filter from its DFT and propose a $\frac{100}{100}$ ³⁵ not be localized simultaneously in time and frequency domain and much simpler approach to obtain the discrete-time UP proposed ¹⁰¹ ³⁶ there exists a lower bound of 0.25 on the TFP [\[5\].](#page--1-0) Chui et al. [\[26\]](#page--1-0) by Sharma et al. [31]. We compare the performance of proposed ¹⁰² ³⁷ determine the similar uncertainty lower bound for band pass func-
UP with that proposed by Slepian [14] and Ishii and Furukawa and Furukawa 103 ³⁸ tions. Venkatesh et al. [\[25\]](#page--1-0) studied the limitations of UP proposed [15] employing the discrete time Fourier transform (DTFT) of the ¹⁰⁴ ³⁹ by Ishii and Furukawa [\[15\].](#page--1-0) They obtained the continuous time sig-
 Filter to measure its fraguery variance. In an other proposed to 105 ⁴⁰ nal from the samples of the symmetric low pass bandlimited signal approach in the paper, the sum of time variance and frequency 106 ⁴¹ by interpolation and removed the inconsistency between the def- variance is used to formulate a positive definite matrix and the 107 ⁴² initions of discrete-time and continuous-time variances. However, eigenvectors of the proposed positive definite matrix are used to 108 ⁴³ the variance expressions proposed by them are applicable to zero sobtain the samples of time–frequency localized bandlimited scal- 109 ⁴⁴ phase low pass bandlimited functions only. The same same same proposed method, we modify the ¹¹⁰ There are various notions in the literature to measure the efphase low pass bandlimited functions only.

 45 Nam [27] formulates the discrete-time measure to quantify the product form of discrete-time uncertainty measure proposed by 111 ⁴⁶ uncertainty in time–frequency analysis using the discrete Fourier Venkatesh et al. [25] to the summation form and use it to design ¹¹² ⁴⁷ transform (DFT) for a subclass of finite length discrete signals. The bandlimited low pass function with TFP close to the lowest ¹¹³ ⁴⁸ They derive the relation between the uncertainties for discrete-
possible lower bound of 0.25. This method simplifies the condition of 1¹⁴ ⁴⁹ time and continuous-time cases. However, they do not determine of regularity of the scaling function, therefore, we further use it for 115 ⁵⁰ the filter or the function optimally localized in time and frequency the design of time-frequency localized three-band filter bank. ¹¹⁶ Nam [\[27\]](#page--1-0) formulates the discrete-time measure to quantify the ized wavelet filter banks. They design the time–frequency localized

¹ calization. In this paper, we propose a novel design method for calized scaling filter for the specified time variance or frequency ⁶⁷ 2 time–frequency localization of a regular scaling filter and the cor- variance of the filter. Sharma et al. [31] use UP for linear oper- $\,$ 68 $\,$ ³ responding scaling function. We show that the proposed method ators to design time–frequency localized filter, however, they do 69 ⁴ outperforms Slepians' [\[14\],](#page--1-0) Ishii and Furukawas' [\[15\]](#page--1-0) methods for not impose the regularity constraint to obtain the time–frequency ⁷⁰ 5 time–frequency localization of discrete scaling filters as well as $\;$ localized scaling filter and function. Bhati et al. [\[16\]](#page--1-0) design the $\;$ $\;$ 7 1 6 72 continuous scaling functions. We use the concentration measures ⁷ for time-limited functions proposed by Bhati et al. [16] and pro- however, the designed scaling filter is not optimally localized with ⁷³ ⁸ pose a design method for the time–frequency localized three-band TFP close to 0.25. Gabors' UP [\[5\]](#page--1-0) for continuous-time functions ⁷⁴ ⁹ biorthogonal linear phase (BOLP) wavelet filter bank of length nine is generally studied assuming function $f(t) \in L_2(\Re)$ [32,33,25,34]. ⁷⁵ 10 and regularity order of one. The design problem of time-frequency Bhati et al. [\[16\]](#page--1-0) emphasize that $tf(t) \in L_2(\mathfrak{M})$ and $\frac{df(t)}{dt} \in L_2(\mathfrak{M})$ to-¹¹ localized wavelet filter banks can be addressed in two different gether implies $f(t) \in L_2(\Re)$. Bhati et al. [35] design time-frequency ⁷⁷ ¹² ways, that directly affects the complexity of the design problem. I localized regular linear phase scaling filters, optimally localized in ⁷⁸ ¹³ Time–frequency localized wavelet filter banks can be designed ei-
13 Time–frequency localized wavelet filter banks can be designed ei-
11 time and frequency domains, however, the proposed semidefinite of ⁷⁹ ¹⁴ ther by time–frequency localization of scaling and wavelet func-

relaxation does not ensure time–frequency localing ⁸⁰ ¹⁵ tions [\[17–19,16\],](#page--1-0) or the filters of a regular perfect reconstruction function generated from the cascade iterations of the designed 81 ¹⁶ filter bank (PRFB) [\[8,20–23\].](#page--1-0) The former deals with Heisenberg un-
¹⁶ filter bank (PRFB) [8,20–23]. The former deals with Heisenberg uncalized scaling filter for the specified time variance or frequency variance of the filter. Sharma et al. [\[31\]](#page--1-0) use UP for linear operators to design time–frequency localized filter, however, they do not impose the regularity constraint to obtain the time–frequency time–frequency localized scaling function as well as scaling filter, is generally studied assuming function $f(t) \in L_2(\Re)$ [\[32,33,25,34\].](#page--1-0) gether implies $f(t) \in L_2(\Re)$. Bhati et al. [\[35\]](#page--1-0) design time–frequency localized regular linear phase scaling filters, optimally localized in time and frequency domains, however, the proposed semidefinite relaxation does not ensure time–frequency localization of scaling function generated from the cascade iterations of the designed scaling filter.

18 on HUP for signals in $L_2(\mathbb{Z})$. The time-frequency localized filters (PSWF) and the HUP are used to design the time-frequency lo-
⁸⁴ ¹⁹ do not ensure that the corresponding scaling and wavelet functions calized scaling filter which generates the time–frequency localized 85 ²⁰ are also well localized in time and frequency domain. Therefore, in scaling function with TFP close to the lowest possible lower bound ⁸⁶ ²¹ this paper, we propose a design method in which we use the ex- of 0.25. In the approach based on PSWF, we maximize the energy 87 ²² pressions for time variance and frequency variance proposed by of the time-limited filter in a specified bandwidth and propose a 88 ²³ Bhati et al. [\[16\]](#page--1-0) and the sum of the weighted TFP's of the scal- design method which ensures time-frequency localization of scal- 89 ²⁴ ing and wavelet functions is minimized with respect to the free ing filter as well as the scaling function. Bhati et al. [16] design 90 ²⁵ parameters of the filter bank. The same that the same of the effequency localized scaling function with unit Sobolev regu-In this paper, the notions of prolate spheroid wave functions (PSWF) and the HUP are used to design the time–frequency localized scaling filter which generates the time–frequency localized scaling function with TFP close to the lowest possible lower bound of 0.25. In the approach based on PSWF, we maximize the energy of the time-limited filter in a specified bandwidth and propose a design method which ensures time–frequency localization of scaling filter as well as the scaling function. Bhati et al. [\[16\]](#page--1-0) design two and design a time–frequency localized scaling function with Sobolev regularity of two. HUP for linear operators is used in the second approach subject to the linear constraints for regularity to design time–frequency localized scaling filter and the function. Sharma et al. [\[31\]](#page--1-0) use the notion of quantum harmonic oscillator and DFT and propose the discrete-time UP which is an equivalent of Gabor's UP for continuous time functions. In this paper, we compute the frequency variance of filter from its DFT and propose a much simpler approach to obtain the discrete-time UP proposed by Sharma et al. [\[31\].](#page--1-0) We compare the performance of proposed UP with that proposed by Slepian $[14]$ and Ishii and Furukawa [\[15\]](#page--1-0) employing the discrete time Fourier transform (DTFT) of the filter to measure its frequency variance. In an another proposed approach in the paper, the sum of time variance and frequency variance is used to formulate a positive definite matrix and the eigenvectors of the proposed positive definite matrix are used to obtain the samples of time–frequency localized bandlimited scal-Venkatesh et al. $[25]$ to the summation form and use it to design the bandlimited low pass function with TFP close to the lowest possible lower bound of 0.25. This method simplifies the condition of regularity of the scaling function, therefore, we further use it for the design of time–frequency localized three-band filter bank.

⁵¹ domains. Parhizkar et al. [\[28\]](#page--1-0) design the time–frequency local-
⁵¹ domains. Parhizkar et al. [28] design the time–frequency local-
 52 ized filter however, the filter is not a regular scaling filter and do fect the performance of the filter bank in signal classification [36]. 118 ⁵³ not generate a time–frequency localized scaling function. Lebedeva Two-band ideal filter banks suffer from poor frequency resolution ¹¹⁹ ⁵⁴ and Prestin [\[29\]](#page--1-0) used the notion of Brietenberg uncertainty con-
⁵⁴ and Prestin [29] used the notion of Brietenberg uncertainty con-
in the low as well as high frequency band with frequency resolu-⁵⁵ stant and proposed the Parseval periodic wavelet frames optimally tion of each band equal to $\Delta\omega = \pi/2$ [37,35]. Two-band wavelet ¹²¹ ⁵⁶ localized in time and frequency domain. However, they do not de-
⁵⁶ localized in time and frequency domain. However, they do not de-
filter banks generate the wavelet basis functions from the succes-57 termine the corresponding time–frequency localized discrete-time sive cascade iterations on the low pass filter branch. Provided the 123 ⁵⁸ regular scaling filter. Sharma et al. [\[20\]](#page--1-0) reduced the design com- conditions for regularity of the filters are satisfied, each successive ¹²⁴ ⁵⁹ plexity associated with the product of variances and proposed cascade iteration on the low pass branch generates smooth wavelet ¹²⁵ ⁶⁰ minimization of sum of variances to design time–frequency local- basis functions to analyze the signals in their low frequency bands. ¹²⁶ 61 ized wavelet filter banks. They design the time–frequency localized Though the low pass and band pass basis functions of the com- 127 62 filter using eigenfilter approach. However, in order to simplify the monly used wavelet filter bank are localized in time and frequency, 128 63 design problem, they impose the unit norm constraint on the half in the cascade iterative structure used in wavelet multiresolution 129 64 of the filter coefficients of the linear phase filter and therefore analysis suffers from the drawback that successive basis functions 130 65 the designed eigenfilters are not optimally localized in time and analyze the low frequency band only and not the high frequency 131 66 frequency domain. Sharma et al. [\[30\]](#page--1-0) design time–frequency lo- components of the signal. Higher number of cascade iterations 132 The spectral characteristics of the basis functions greatly affect the performance of the filter bank in signal classification [\[36\].](#page--1-0) Two-band ideal filter banks suffer from poor frequency resolution in the low as well as high frequency band with frequency resolution of each band equal to $\Delta \omega = \pi/2$ [\[37,35\].](#page--1-0) Two-band wavelet filter banks generate the wavelet basis functions from its successive cascade iterations on the low pass filter branch. Provided the conditions for regularity of the filters are satisfied, each successive cascade iteration on the low pass branch generates smooth wavelet basis functions to analyze the signals in their low frequency bands. monly used wavelet filter bank are localized in time and frequency, the cascade iterative structure used in wavelet multiresolution analysis suffers from the drawback that successive basis functions analyze the low frequency band only and not the high frequency components of the signal. Higher number of cascade iterations

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