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A novel approach for time–frequency localization of scaling functions and design of three-band biorthogonal linear phase wavelet filter banks

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ABSTRACT

Design of time–frequency localized filters and functions is a classical subject in the field of signal processing. Gabors' uncertainty principle states that a function cannot be localized in time and frequency domain simultaneously and there exists a nonzero lower bound of 0.25 on the product of its time variance and frequency variance called time–frequency product (TFP). Using arithmetic mean (AM)–geometric mean (GM) inequality, product of variances and sum of variances can be related and it can be shown that sum of variances has lower bound of one. In this paper, we compute the frequency variance of the filter from its discrete Fourier transform (DFT) and propose an equivalent summation based discrete-time uncertainty principle which has the lower bound of one. We evaluate the performance of the proposed discrete-time time–frequency uncertainty measure in multiresolution setting and show that the proposed DFT based concentration measure generate sequences which are even more localized in time and frequency domain than that obtained from the Slepian, Ishii and Furukawa's concentration measures. The proposed design approach provides the flexibility in which the TFP can be made arbitrarily close to the lowest possible lower bound of 0.25 by increasing the length of the filter. In the other proposed approach, the sum of the time variance and frequency variance is used to formulate a positive definite matrix to measure the time–frequency joint localization of a bandlimited function from its samples. We design the time–frequency localized bandlimited low pass scaling and band pass wavelet functions using the eigenvectors of the formulated positive definite matrix. The samples of the time–frequency localized bandlimited function is obtained from the eigenvector of the positive definite matrix corresponding to its minimum eigenvalue. The TFP of the designed bandlimited scaling and wavelet functions are close to the lowest possible lower bound of 0.25 and 2.25 respectively. We propose a design method for time–frequency localized three-band biorthogonal linear phase (BOLP) wavelet perfect reconstruction filter bank (PRFB) wherein the free parameters can be optimized for time–frequency localization of the synthesis basis functions for the specified frequency variance of the analysis scaling function. The performance of the designed filter bank is evaluated in classification of seizure and seizure-free EEG signals. It is found that the proposed filter bank outperforms other existing methods for the classification of seizure and seizure-free EEG signals.

1. Introduction

In the last two decades, wavelet filter banks have played a key role in describing nonstationary signals such as speech, seismic, radar, electroencephalogram (EEG), electrocardiogram (ECG) [1–3]. The basis functions generated by the wavelet filter bank provide multiple resolutions in time domain and frequency domain. Time–frequency localized wavelet bases are desirable in signal analysis [4–6], image coding [7–9,8,10], edge detection and image segmentation [11,12]. The joint time–frequency localization of the wavelet basis functions is generally represented by the dimensions of the tile on the time–frequency plane [13] that represents them. The area of the tile or the time–frequency product (TFP) represents the joint localization of the basis function in the time and frequency domain. Various signal processing applications require trade-off in time localization and frequency localization. For example, time localized bases preserve the edges in the image whereas bandwidth compression of an image can be achieved with good frequency lo-

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calization. In this paper, we propose a novel design method for time–frequency localization of a regular scaling filter and the corresponding scaling function. We show that the proposed method outperforms Slepians' [14], Ishii and Furukawas' [15] methods for time–frequency localization of discrete scaling filters as well as continuous scaling functions. We use the concentration measures for time-limited functions proposed by Bhati et al. [16] and propose a design method for the time–frequency localized three-band biorthogonal linear phase (BOLP) wavelet filter bank of length nine and regularity order of one. The design problem of time–frequency localized wavelet filter banks can be addressed in two different ways, that directly affects the complexity of the design problem. Time–frequency localized wavelet filter banks can be designed either by time–frequency localization of scaling and wavelet functions [17–19,16], or the filters of a regular perfect reconstruction filter bank (PRFB) [8,20–23]. The former deals with Heisenberg uncertainty principle (HUP) for signals in $L_2(\mathbb{R})$ and the later is based on HUP for signals in $L_2(\mathbb{Z})$. The time–frequency localized filters do not ensure that the corresponding scaling and wavelet functions are also well localized in time and frequency domain. Therefore, in this paper, we propose a design method in which we use the expressions for time variance and frequency variance proposed by Bhati et al. [16] and the sum of the weighted TFP's of the scaling and wavelet functions is minimized with respect to the free parameters of the filter bank.

There are various notions in the literature to measure the effective support of a function in time or frequency domain [24,14,5,15,25]. Slepian et al. [14] measures the energy concentration in the given frequency band for time-limited signals whereas, Gabor uses the notion of variances to measure the effective supports in time and frequency domains [5]. Time–frequency uncertainty principle (UP) in discrete domain is studied by several authors in the literature. Ishii and Furukawa [15] proposed time–frequency UP for discrete time sequences. Gabor showed that signals in $L_2(\Re)$ cannot be localized simultaneously in time and frequency domain and there exists a lower bound of 0.25 on the TFP [5]. Chui et al. [26] determine the similar uncertainty lower bound for band pass functions. Venkatesh et al. [25] studied the limitations of UP proposed by Ishii and Furukawa [15]. They obtained the continuous time signal from the samples of the symmetric low pass bandlimited signal by interpolation and removed the inconsistency between the definitions of discrete-time and continuous-time variances. However, the variance expressions proposed by them are applicable to zero phase low pass bandlimited functions only.

Nam [27] formulates the discrete-time measure to quantify the uncertainty in time–frequency analysis using the discrete Fourier transform (DFT) for a subclass of finite length discrete signals. They derive the relation between the uncertainties for discrete-time and continuous-time cases. However, they do not determine the filter or the function optimally localized in time and frequency domains. Parhizkar et al. [28] design the time–frequency localized filter however, the filter is not a regular scaling filter and do not generate a time–frequency localized scaling function. Lebedeva and Prestin [29] used the notion of Brietenberg uncertainty constant and proposed the Parseval periodic wavelet frames optimally localized in time and frequency domain. However, they do not determine the corresponding time–frequency localized discrete-time regular scaling filter. Sharma et al. [20] reduced the design complexity associated with the product of variances and proposed minimization of sum of variances to design time–frequency localized wavelet filter banks. They design the time–frequency localized filter using eigenfilter approach. However, in order to simplify the design problem, they impose the unit norm constraint on the half of the filter coefficients of the linear phase filter and therefore the designed eigenfilters are not optimally localized in time and frequency domain. Sharma et al. [30] design time–frequency localized scaling filter for the specified time variance or frequency variance of the filter. Sharma et al. [31] use UP for linear operators to design time–frequency localized filter, however, they do not impose the regularity constraint to obtain the time–frequency localized scaling filter and function. Bhati et al. [16] design the time–frequency localized scaling function as well as scaling filter, however, the designed scaling filter is not optimally localized with TFP close to 0.25. Gabors' UP [5] for continuous-time functions is generally studied assuming function $f(t) \in L_2(\Re)$ [32,33,25,34]. Bhati et al. [16] emphasize that $tf(t) \in L_2(\Re)$ and $\frac{df(t)}{dt} \in L_2(\Re)$ together implies $f(t) \in L_2(\Re)$. Bhati et al. [35] design time–frequency localized regular linear phase scaling filters, optimally localized in time and frequency domains, however, the proposed semidefinite relaxation does not ensure time–frequency localization of scaling function generated from the cascade iterations of the designed scaling filter.

In this paper, the notions of prolate spheroid wave functions (PSWF) and the HUP are used to design the time–frequency localized scaling filter which generates the time–frequency localized scaling function with TFP close to the lowest possible lower bound of 0.25. In the approach based on PSWF, we maximize the energy of the time-limited filter in a specified bandwidth and propose a design method which ensures time–frequency localization of scaling filter as well as the scaling function. Bhati et al. [16] design time–frequency localized scaling function with unit Sobolev regularity. In this work, we use a scaling filter with regularity order of two and design a time–frequency localized scaling function with Sobolev regularity of two. HUP for linear operators is used in the second approach subject to the linear constraints for regularity to design time–frequency localized scaling filter and the function. Sharma et al. [31] use the notion of quantum harmonic oscillator and DFT and propose the discrete-time UP which is an equivalent of Gabor's UP for continuous time functions. In this paper, we compute the frequency variance of filter from its DFT and propose a much simpler approach to obtain the discrete-time UP proposed by Sharma et al. [31]. We compare the performance of proposed UP with that proposed by Slepian [14] and Ishii and Furukawa [15] employing the discrete time Fourier transform (DTFT) of the filter to measure its frequency variance. In an another proposed approach in the paper, the sum of time variance and frequency variance is used to formulate a positive definite matrix and the eigenvectors of the proposed positive definite matrix are used to obtain the samples of time–frequency localized bandlimited scaling and wavelet functions. In the proposed method, we modify the product form of discrete-time uncertainty measure proposed by Venkatesh et al. [25] to the summation form and use it to design the bandlimited low pass function with TFP close to the lowest possible lower bound of 0.25. This method simplifies the condition of regularity of the scaling function, therefore, we further use it for the design of time–frequency localized three-band filter bank.

The spectral characteristics of the basis functions greatly affect the performance of the filter bank in signal classification [36]. Two-band ideal filter banks suffer from poor frequency resolution in the low as well as high frequency band with frequency resolution of each band equal to $\Delta\omega = \pi/2$ [37,35]. Two-band wavelet filter banks generate the wavelet basis functions from its successive cascade iterations on the low pass filter branch. Provided the conditions for regularity of the filters are satisfied, each successive cascade iteration on the low pass branch generates smooth wavelet basis functions to analyze the signals in their low frequency bands. Though the low pass and band pass basis functions of the commonly used wavelet filter bank are localized in time and frequency, the cascade iterative structure used in wavelet multiresolution analysis suffers from the drawback that successive basis functions analyze the low frequency band only and not the high frequency components of the signal. Higher number of cascade iterations

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