ARTICLE IN PRESS
YDSPR:1963

Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

# Mutual coupling effect and compensation in non-uniform arrays for direction-of-arrival estimation

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ARTICLE INFO

*Article history:*
Available online xxxx

*Keywords:*
Direction finding
Mutual coupling
Non-uniform arrays
Direction-of-arrival
Nested array
Co-prime arrays

ABSTRACT

In this paper, we investigate the effect of mutual coupling on direction-of-arrival (DOA) estimation using non-uniform arrays. We compare and contrast the DOA estimation accuracy in the presence of mutual coupling for three different non-uniform array geometries, namely, minimum redundancy arrays (MRAs), nested arrays, and co-prime arrays, and for two antenna types, namely dipole antennas and microstrip antennas. We demonstrate through numerical simulations that the mutual coupling, if unaccounted for, can, in general, lead to performance degradation, with the MRA faring better against mutual coupling than the other two non-uniform structures for both antenna types. We also propose two methods that can compensate for the detrimental effects of mutual coupling, leading to accurate and reliable DOA estimation. Supporting numerical simulation results are provided which show the effectiveness of the proposed compensation methods.



## 1. Introduction

Antenna arrays are employed for direction-of-arrival (DOA) estimation in a broad range of applications including radar, sonar, and wireless communications [1–3]. High-resolution DOA estimation techniques, such as MUSIC [4], ESPRIT [5], and $\ell_1$-SVD [6], are widely used for direction finding. In real antenna arrays, these techniques, in their original implementations, suffer from a model mismatch which, among other factors, can be attributed to mutual coupling between the elements. Mutual coupling occurs when an external illuminating source induces a current on the surface of each array element, causing it to radiate. A portion of the radiated signal is captured by the remaining elements in the array. If unaccounted for, this interaction affects the characteristics and the performance of the array [7,8].

The mutual coupling between the array elements can be captured in a matrix called the mutual coupling matrix (MCM). Two major trends exist in the literature for performing DOA estimation in the presence of mutual coupling. The first deals with the case of perfectly known or modeled MCM, wherein the DOA estimation procedure is modified to account for the coupling [9]. In the second trend, the MCM is assumed to be unknown or imprecisely known with a specific structure, and is jointly estimated along with the source directions.

Electromagnetic theory and numerical or analytical modeling techniques are typically employed to characterize the MCM [8, 10–14]. The MCM depends on the self and mutual impedances between the array elements. One of the earliest methods that model the coupling matrix is the open-circuit method [8]. This method treats the array as a bilateral terminal network and relates the uncoupled voltages with the coupled voltages through a mutual impedance matrix. For dipole antennas, the elements in the mutual impedance matrix can be approximated by closed-form expressions [15]. An extension of the open-circuit method has been proposed in [10], where two types of mutual impedances are defined, namely, the transmission mutual impedance and the re-radiation mutual impedance. In [11], the receiving-mutual-impedance method (RMIM) is described for use in receive-only antenna arrays. As such, it provides a more accurate coupling model in DOA estimation applications. RMIM considers each antenna pair separately to compute the receiving mutual impedances. An enhancement of RMIM is presented in [12], which takes into account all the elements simultaneously in order to compute the receiving mutual impedances.

For a perfectly known or modeled MCM, DOA estimation algorithms can be modified to incorporate the coupling and compensate for it in order to achieve accurate source directions [9]. However, if the modeled MCM is not exact, the performance of the DOA estimation is degraded. Moreover, the MCM must be recalibrated periodically to account for any changes in local condi-

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http://dx.doi.org/10.1016/j.dsp.2016.06.005
1051-2004/

tions. For instance, the presence of a new scatterer in the vicinity of the antenna array changes the mutual coupling. Several methods have been proposed to circumvent these issues. These methods assume the coupling matrix to be unknown or imprecisely known and aim to jointly estimate the MCM along with the source DOAs [7,16,17]. Ref. [7] presents an iterative method to estimate the MCM, the DOAs, and the antenna gains, wherein the cost function is minimized with respect to one unknown quantity at a time while keeping the remaining two unknowns fixed. A maximum likelihood estimator for DOA estimation under unknown multipath and unknown mutual coupling has been proposed in [16]. Ref. [17] employs sparse reconstruction to perform DOA estimation in the presence of unknown mutual coupling. However, all of these aforementioned methods have been developed for uniform linear arrays (ULAs) and take advantage of the special structure of the corresponding MCMs. Although these methods can be modified and applied to non-uniform arrays, they fail to take advantage of the increased degrees-of-freedom (DOFs) offered by non-uniform arrays for DOA estimation [18–22]. Recall that an $N_A$-element non-uniform array can provide $O(N_A^2)$ DOFs, thereby permitting DOA estimation of more sources than sensors. An iterative method for DOA estimation using non-uniform arrays in the presence of mutual coupling was proposed in [23]. This method treats the non-uniform array as a subset of a ULA and, therefore, cannot take full advantage of the increased DOFs.

In this paper, we investigate the mutual coupling effect in non-uniform arrays. First, we examine the impact of coupling on the DOA estimation accuracy for different array geometries, including minimum redundancy arrays (MRA) [18], nested arrays [20], and co-prime arrays [21,22]. The performance is evaluated for different array sizes and for two antenna element types, namely, dipole antenna and microstrip antenna. The latter is becoming increasingly popular in radar and wireless communications due to its low profile, ease of fabrication, low cost, and compatibility with radio frequency (RF) circuit boards. A computational electromagnetics software package, FEKO [24], is used to model the antenna arrays, and the RMIM [12] is used to compute the coupling matrices based on the obtained measurements. We show that the MRA provides superior performance compared to the nested and co-prime geometries, irrespective of the antenna type. Second, we propose two compensation methods that allow accurate DOA estimation using non-uniform arrays in the presence of mutual coupling. The first method assumes partial knowledge of the mutual coupling and employs an iterative approach to update the perturbed MCM and DOAs. Sparse signal reconstruction is used to find the source directions for a given coupling matrix, and a global optimization algorithm called covariance matrix adaptation evolution strategy (CMA-ES) [25] is used to update the MCM while keeping the DOAs fixed. The second method assumes unknown coupling and simultaneously estimates the MCM, the source powers, and sources directions by minimizing a cost function using CMA-ES. Finally, the effectiveness of the proposed methods is evaluated through numerical examples.

The remainder of this paper is organized as follows. High-resolution DOA estimation using non-uniform arrays is briefly reviewed in Section 2. The signal model in the presence of mutual coupling is also presented in the same section. In Section 3, DOA estimation performance of different non-uniform array geometries is evaluated and compared for the case of uncompensated mutual coupling. Section 4 discusses the two proposed compensation methods that allow accurate DOA estimation under mutual coupling and provides supporting numerical results. Section 5 concludes the paper.

## 2. DOA estimation using non-uniform arrays

A general $N_A$-element linear array is considered. The elements positions are assumed to be integer multiples of the unit spacing, i.e., $x_i = n_i d_0$, $i = 1, \ldots, N_A$, where $x_i$ is the position of the $i$th array element, $n_i$ is an integer, and $d_0$ is the unit spacing which is usually set to half-wavelength at the operating frequency. Assume that $D$ narrowband sources with directions $\{\theta_1, \theta_2, \ldots, \theta_D\}$ and powers $\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_D^2\}$ impinge on the array, where $\theta$ is measured relative to broadside. In the absence of mutual coupling, the received data vector at snapshot $t$ can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{s}(t)$ is the $D \times 1$ source signal vector, $\mathbf{n}(t)$ is the $N_A \times 1$ noise vector, and $\mathbf{A}$ is the $N_A \times D$ array manifold matrix whose $(i, d)$th element is given by

$$[\mathbf{A}]_{i,d} = \exp(jk_0 x_i \sin\theta_d). \tag{2}$$

Here, $k_0$ is the wavenumber at the operating frequency and $\theta_d$ is the DOA of the $d$th source. Under the assumptions of uncorrelated sources and spatially and temporally white noise, the covariance matrix can be expressed as

$$\mathbf{R}_{xx} = E\left\{\mathbf{x}(t)\mathbf{x}(t)^H\right\} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_n^2\mathbf{I}, \tag{3}$$

where $E\{\cdot\}$ is the expectation operator, $(\cdot)^H$ denotes conjugate transpose, $\mathbf{R}_{ss} = \mathrm{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_D^2\}$ is the source covariance matrix, $\sigma_n^2$ is the noise variance, and $\mathbf{I}$ is an $N_A \times N_A$ identity matrix.

Two approaches can be used for DOA estimation. The first approach is based on covariance matrix augmentation [26–28], while the second uses spatial smoothing [21,22]. Since the augmented covariance matrix in the first approach may not always be positive semidefinite, we consider spatial smoothing based approach in this paper, which is briefly reviewed below.

Vectorizing the covariance matrix in (3), we obtain

$$\mathbf{z} = \mathrm{vec}\{\mathbf{R}_{xx}\} = \tilde{\mathbf{A}}\mathbf{p} + \sigma_n^2\tilde{\mathbf{i}}, \tag{4}$$

where $\mathbf{p} = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_D^2]^T$ is the source powers vector, $\tilde{\mathbf{A}} = \mathbf{A}^* \odot \mathbf{A}$, the symbol '$\odot$' denotes the Khatri–Rao product, the superscript '$*$' denotes complex conjugate, and $\tilde{\mathbf{i}} = \mathrm{vec}\{\mathbf{I}\}$ is the vectorized identity matrix. The vector $\mathbf{z}$ emulates measurements at a longer array whose elements positions are given by the difference coarray of the non-uniform arrays, while the $N_A^2 \times D$ matrix $\tilde{\mathbf{A}}$ is the corresponding manifold matrix [29]. Assuming that the difference coarray has contiguous elements between $-Ld_0$ and $+Ld_0$, the data measurements can be rearranged to form a new $(2L+1) \times 1$ vector $\mathbf{z}_f$, which contains measurements at these positions,

$$\mathbf{z}_f = \tilde{\mathbf{A}}_f\mathbf{p} + \sigma_n^2\tilde{\mathbf{i}}_f. \tag{5}$$

Since the sources are replaced by their powers in (5) and the noise is deterministic, the sources now appear as coherent, and subspace-based high-resolution methods can no longer be applied directly to perform DOA estimation. Spatial smoothing is used to build the rank of the covariance matrix of $\mathbf{z}_f$ [30]. The filled part of the difference coarray is divided into $(L+1)$ overlapping subarrays, each having $(L+1)$ contiguous elements. The positions of the elements of the $m$th subarray are given by the following set

$$\left\{(l+1-m)d_0, l = 0, 1, \ldots, L\right\}. \tag{6}$$

The received data vector at the $m$th subarray is denoted by $\mathbf{z}_{f_m}$, and the spatially smoothed covariance matrix is then computed as

$$\overline{\mathbf{R}}_{zz} = \frac{1}{L+1}\sum_{m=1}^{L+1}\mathbf{z}_{f_m}\mathbf{z}_{f_m}^H. \tag{7}$$

Please cite this article in press as: E. BouDaher et al., Mutual coupling effect and compensation in non-uniform arrays for direction-of-arrival estimation, Digit. Signal Process. (2016), http://dx.doi.org/10.1016/j.dsp.2016.06.005