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Sparse source localization using perturbed arrays via bi-affine modeling

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ABSTRACT

Non-uniform spatial sampling geometries, such as nested and coprime arrays, are provably capable of localizing $O(M^2)$ sources using only M sensors. However, such guarantees require the physical locations of the sensors to satisfy certain constraints, as dictated by the corresponding array geometries. In this paper, we consider the scenario when these constraints may be violated, leading to unknown perturbations on the locations of sensors. Such perturbations can have detrimental effect on the performance of virtual array based direction-of-arrival (DOA) estimation algorithms, since the perturbed virtual array will no longer be a uniform linear array (ULA). We propose a novel self-calibration approach for underdetermined DOA estimation with such arrays, that makes extensive use of the redundancies (or repeated elements) in the virtual array. Assuming small perturbations, and a sparse grid-based model for the DOAs, we extract a novel "bi-affine" model (affine in the perturbation variable, and linear in the source powers) from the covariance matrix of the received signals. The redundancies in the co-array are then exploited to eliminate the nuisance perturbation variable, and reduce the bi-affine problem to a linear underdetermined (sparse) problem in source powers, from which the DOAs can be exactly recovered under suitable conditions. This reduction is derived for both ULA and a newly-introduced robust version of coprime arrays, when the covariance matrix of the received signals is exactly known. Our approach is compared and contrasted with recently developed algorithms for blind gain and phase calibration (BGPC), whose signal model is fundamentally different from ours. We also provide an iterative algorithm to jointly solve for the DOAs and perturbation values when we can only estimate the covariance matrix using a finite number of snapshots.

1. Introduction

Directions-of-arrival (DOA) estimation of energy emitting sources using sensor arrays finds important application in problems ranging from target localization in radar system to speech enhancement using microphone arrays. In recent times, new sparse array geometries, such as coprime [1] and nested arrays [2], have been proposed that are capable of identifying $O(M^2)$ sources using just M sensors, exploiting the enhanced degrees of freedom offered by their difference co-arrays (or, virtual arrays) [3]. In order to exploit the enhanced degrees of freedom, so far, two main approaches for DOA estimation have been proposed: 1) Subspace methods, 2) Sparsity based methods. In the former approach, which is based on the MUSIC algorithm, the subspace properties of the spatially smoothed co-array manifold is used to estimate the DOAs [4]. However, in the latter approach, the range of all possible directions is discretized into a grid, and then the DOA estimation problem is reformulated as a sparse representation problem [5–8]. We review this approach in more details in Section 2.

It is well known that array imperfections such as gain and/or phase error, perturbations in sensor locations, and mutual coupling, can significantly degrade the performance of DOA estimation algorithms [9,10]. This is mainly due to the strong dependence of these algorithms on the accurate knowledge of the underlying array manifold. In this paper, we consider the sensor location error as the only imperfection associated with the physical array, i.e., we assume that the sensor locations are perturbed from their nominal positions. The problem of DOA estimation using such perturbed arrays has been well studied for more than two decades. Existing approaches mostly treat the perturbations as unknown but deterministic parameters, and then estimate these parameters jointly with the DOAs. Classical methods such as [10–13], resolve array uncertainties using eigenstructure-based methods, or variants of the maximum-likelihood approach. Recently, [14] proposed

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a unified framework for different kind of array imperfections, and proposed a Bayesian approach for array calibration and DOA estimation. However, these approaches mostly work for an overdetermined signal model (fewer sources than sensors), primarily because many of them consider a uniform linear array.

In recent times, the problem of blind gain and phase calibration (BGPC) has been formulated as a bilinear problem [15], which in turn, can be recast as a convex optimization problem, using the idea of "lifting" [16–18]. However, such a formulation does not consider the concept of co-array, and, hence their guarantees are not applicable for an underdetermined signal model where the number of sources can possibly be $O(M^2)$.

In contrast, the authors in [19], studied the effect of co-array geometry on the BGPC problem and proposed a new self-calibration algorithm for nested arrays in presence of gain/phase errors. Their approach builds on and extends the method in [12], which was originally proposed for a ULA. However, in this paper, we consider perturbations in sensor locations, which gives rise to a signal model, which is distinctly different from that considered in [19]. In BGPC problems, the gain and/or phase of the sensors are unknown, and the goal is to resolve both unknown gain and/or phase and the DOAs. In our case, we assume that the phase and gain of the signals received from the sensors are ideal, but the location of the sensors are perturbed. We will compare the signal model defined for gain/phase error, which has been studied in [19], against sensor location error in Section 2.2, and establish important differences between them.

Since the self calibration algorithm developed in [19] cannot be directly applied to our case, we follow a different approach in this paper. We assume that the perturbations are small, so that we can approximate the coarray manifold using its first order Taylor series expansion. This formulation leads to a "bi-affine" model, which is linear in source powers, and affine in the perturbation variable. We show that it is possible to recover the DOAs even in presence of the nuisance perturbation variables, via a clever elimination of variables. By exploiting the pattern of repeating elements, it is possible to reduce the said bi-affine problem to a linear underdetermined (sparse) problem in source powers, which can be efficiently solved using ℓ_1 minimization. We establish precise conditions under which such reduction is possible, for both ULA and a robust version of coprime arrays.

The paper is organized as follows. In Sec. 2 we compare and contrast different kinds of array imperfections (gain/phase error, sensor location perturbation) in terms of their effects on the difference co-array. In Sec. 3, we introduce the bi-affine model for DOA estimation with perturbed sensors. Sec. 4, establishes a transformation under which we can write the bi-affine problem as a linear problem in source powers, via elimination of the unknown perturbation variable. The specific details of this transformation depend on the array geometry. In Sec. 5, we review an iterative algorithm proposed in [20] to jointly solve for DOAs and source powers when we only have an estimate of the covariance matrix. Numerical simulations are conducted in Sec. 6. Sec. 7 concludes the paper.

Notation: Throughout this paper, matrices are represented by upper case bold letters, and vectors by lower case bold letters. The symbol x_i represents the ith entry of a vector $\mathbf{x}$. The symbol $\jmath$ denotes the imaginary unit $\sqrt{-1}$. The symbols $(.)^*, (.)^T, (.)^H$ stand for the conjugate, transpose, and hermitian, respectively. The symbols $\circ, \odot, \otimes$ represent the Hadamard product, Khatri–Rao product, and Kronecker product, respectively. The symbol $\|.\|_F$ denotes the matrix Frobenius norm and vec(.) represents the vectorized form of a matrix.

2. Signal model for gain/phase error vs location errors

Consider a linear array of M antennas impinged by K narrowband sources with unknown directions of arrival (DOA) $\boldsymbol{\theta} \in \mathbb{R}^K$, $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]^T$. Let $\mathbf{y}[l] \in \mathbb{C}^M$ be the vector of signals received by the M antennas, $\mathbf{x}[l] \in \mathbb{C}^K$ represent the emitted signals from K sources, and $\mathbf{w}[l]$ be the additive noise (all corresponding to the lth time snapshot). The source signals are assumed to be zero mean, and pairwise uncorrelated, and the noise vector is zero mean, i.i.d. with variance σ_w^2, and uncorrelated from the signal. We do not make any specific assumptions on the distribution of the signal or noise.

The sensors are designed to be at the nominal locations $\tilde{d}_1, \tilde{d}_2, \cdots, \tilde{d}_M$, where $\tilde{d}_m \in \mathbb{R}$ for $1 \le m \le M$, and $\tilde{d}_m = Dd_m$. Here, $d_m \in \mathbb{Z}$, and D is the minimum inter-element spacing of the array, which is typically chosen to be $D = \lambda/2$, λ being the carrier wavelength of the narrowband sources. Note that d_m are the normalized sensor locations (and $\tilde{d}_m$ are the actual sensor locations). In the sequel, we will use the normalized locations as we introduce the perturbed array model. In this paper, we consider two different array geometries: uniform linear array (ULA), and coprime array. In a ULA, we have $d_m = m - 1$, for $m = 1, \cdots, M$. A coprime array, however, is comprised of two different ULAs with spacings N_1 and N_2, where N_1 and N_2 are coprime numbers. We will review the coprime arrays in more detail in Sec. 2.1.2. To simplify the notations, we designate a spatial frequency $\omega_i = \frac{2\pi D}{\lambda}\sin\theta_i$ corresponding to each direction of arrival θ_i for $1 \le i \le K$. Choosing $D = \lambda/2$, we have $\omega_i = \pi \sin\theta_i$. Also, let $\boldsymbol{\omega} = [\omega_1, \cdots, \omega_K]^T$ be the vector of spatial frequencies associated with the K sources.

Let $\boldsymbol{\zeta} \in \mathbb{C}^M$ be a vector of unknown parameters associated with array imperfections, such as gain/phase, or sensor location errors. The received samples at the time instant l can be written as

$$\mathbf{y}[l] = \mathbf{A}(\boldsymbol{\omega}, \boldsymbol{\zeta})\mathbf{x}[l] + \mathbf{w}[l] \tag{1}$$

in which $\mathbf{A}(\boldsymbol{\omega}, \boldsymbol{\zeta}) = [\mathbf{a}(\omega_1, \boldsymbol{\zeta}), \ldots, \mathbf{a}(\omega_K, \boldsymbol{\zeta})]$ denotes the array manifold, and $\mathbf{a}(\omega_i, \boldsymbol{\zeta}) \in \mathbb{C}^M$ is the steering vector for the ith source. In the absence of array imperfections ($\boldsymbol{\zeta} = \mathbf{0}$), the mth element of the steering vector corresponding to direction θ_i is given by $a_m(\omega_i, \mathbf{0}) = e^{\jmath d_m \omega_i}$. In the following subsections, we will first review the concept of a virtual array by considering the covariance matrix for the unperturbed problem [2]. Subsequently, we will discuss and distinguish the signal models corresponding to two different kinds of array imperfections: (i) gain/phase error, and (ii) sensor location error.

2.1. Virtual array in the absence of array imperfections

In the absence of array imperfections ($\boldsymbol{\zeta} = \mathbf{0}$), we can write the covariance matrix of the received signals as

$$\mathbf{R_y} = E(\mathbf{y}\mathbf{y}^H) = \mathbf{A}_0(\boldsymbol{\omega})\mathbf{R_x}\left(\mathbf{A}_0(\boldsymbol{\omega})\right)^H + \sigma_w^2\mathbf{I} \tag{2}$$

where $\mathbf{R_x} = E(\mathbf{x}\mathbf{x}^H)$ is the covariance matrix of the sources, and $\mathbf{A}_0(\boldsymbol{\omega}) = \mathbf{A}(\boldsymbol{\omega}, \mathbf{0})$. Assuming that the sources are uncorrelated, i.e., $\mathbf{R_x}$ is diagonal, following [2] the vectorized form of the covariance matrix can be written as

$$\mathbf{z} = \mathbf{A}_{\mathrm{KR},0}(\boldsymbol{\omega})\tilde{\mathbf{p}} + \sigma_w^2\,\mathrm{vec}(\mathbf{I}), \tag{3}$$

where $\mathbf{A}_{\mathrm{KR},0}(\boldsymbol{\omega}) = \mathbf{A}_0(\boldsymbol{\omega})^* \odot \mathbf{A}_0(\boldsymbol{\omega})$ is the difference co-array, $\tilde{\mathbf{p}} = [p_1, p_2, \cdots, p_K]$ is the diagonal of $\mathbf{R_x}$, and $\mathbf{z} = \mathrm{vec}(\mathbf{R_y})$. The $(m + (m' - 1)M, i)$-th element of $\mathbf{A}_{\mathrm{KR},0}(\boldsymbol{\theta})$ is given by $e^{\jmath\omega_i(d_m - d_{m'})}$. Therefore, each column of $\mathbf{A}_{\mathrm{KR},0}(\boldsymbol{\omega})$ is characterized by the difference co-array:

$$S_{\mathrm{ca}} = \{d_m - d_{m'}, 1 \le m, m' \le M\}$$