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An iterative approach for sparse direction-of-arrival estimation in co-prime arrays with off-grid targets ☆

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ABSTRACT

This paper addresses the problem of direction of arrival (DOA) estimation by exploiting the sparsity enforced recovery technique for co-prime arrays, which can increase the degrees of freedom. To apply the sparsity based technique, the discretization of the potential DOA range is required and every target must fall on the predefined grid. Off-grid target can highly deteriorate the recovery performance. To the end, this paper takes the off-grid DOAs into account and reformulates the sparse recovery problem with unknown grid offset vector. By introducing a convex function majorizing the given objective function, an iterative approach is developed to gradually amend the offset vector to achieve final DOA estimation. Numerical simulations are provided to verify the effectiveness of the proposed method in terms of detection ability, resolution ability and root mean squared estimation error, as compared to the other state-of-the-art methods.

1. Introduction

The problem of direction-of-arrival (DOA) estimation has arisen in various applications, such as radar, sonar, radio astronomy and so on [1]. It is well known that for traditional linear array with N sensors, the commonly used subspace based methods [2,3] can resolve up to $N-1$ sources. To detect more sources, new non-uniform linear array geometries, such as nested array [4] and co-prime array [5–8], have been recently proposed. For the non-uniform arrays, two main approaches can be utilized to enhance the degrees of freedom (DOFs), i.e., covariance vectorization [6] and covariance fitting [9]. By vectorizing the covariance matrix of the received signals, a virtual difference coarray with a wider aperture is formed to achieve the extended DOFs. By covariance fitting, the covariance matrix for a redundancy array is recovered according to its Hermitian Toeplitz structure and the extended DOFs can be obtained. With the extended DOFs, the nested array structure in [4] can resolve $\mathcal{O}(N^2)$ sources with only N sensors. However, due to some closely located sensors, the nested array suffers from the mutual coupling problem. The co-prime array structure [5] can address this problem. Such arrays consist of two uniform linear subarrays with M and N sensors, and their corresponding inter-element spacings are $N\lambda/2$ and $M\lambda/2$, respectively, where λ is the wavelength. The co-prime array can resolve $\mathcal{O}(MN)$ sources with $M+N-1$ sensors. To further enhance the DOFs, another co-prime array structure was proposed in [6] by doubling the number of sensors in one subarray, where a larger number of consecutive virtual sensors can be achieved.

Various methods have been proposed to exploit the increased DOFs of co-prime arrays for DOA estimation. In [6], a subspace-based spatial smoothing MUSIC (SS-MUSIC) algorithm was implemented and showed that an increased number of sources can be detected. The SS-MUSIC requires the knowledge of number of sources, to this end, a MUSIC-like subspace method was proposed in [10], where the number of sources is revealed as a by-product of a low-rank denoising stage. However, both the MUSIC-like methods in [6,10] require a consecutive difference coarray lag and the application of spatial smoothing essentially halves the obtained virtual array aperture [11]. Thus the detection performance is compromised. By taking advantages of the fact that the spatial signal spectra are sparse, sparsity-based estimation methods [11–15] have been recently proposed to overcome these disadvantages of the MUSIC-like methods. These sparsity-based techniques

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discretize the range of interest into a grid and assume that the locations of the sources must fall on the predefined grid. However, no matter how fine the grid is, true DOAs are unlikely to lie on the pre-specified grid and off-grid problem can lead to mismatches in the model. The recovery performance is then deteriorated as a result.

To address the off-grid issue, the joint sparsity between original signal and mismatch parameter is exploited in [16,17] and leads to an improved performance over traditional sparsity-based methods. Utilizing the fact that the off-grid DOA can be well approximated by the closest two neighboring grids, an efficient method is proposed in [18] for a single source and multiple well-separated sources. Through linearization, the off grid is modeled in [19] and solved in a Bayesian approach to exploit the joint sparsity among different snapshots. Iterative reweighted algorithms are proposed in [20,21] for joint parameter learning and sparse signal recovery, which mainly applies to uniformly separated cases. However, most of the existing methods focus on traditional linear array and ignore the increased DOFs provided by the difference coarray of co-prime array. To this end, in [22], by using the first-order Taylor approximation, the grid mismatches can be estimated simultaneously with the original signal for co-prime arrays. Recently, grid-less based sparse methods have also been proposed in [9,10]. Due to no need for discretization, these methods can solve the off-grid issue naturally. However, the covariance fitting in [9] requires that the array is a redundancy array, i.e., its difference coarray forms a ULA. For non-redundancy arrays, a ULA subset needs to be selected from the coarray and the extended DOFs are reduced as a result. Meanwhile, the application of spatial smoothing in [10] halves the extended DOFs. Therefore, the achieved DOFs are not fully utilized in these literatures.

In this paper, we address the problem of DOA estimation for non-uniform co-prime arrays with off-grid mismatch, where the increased DOFs are fully exploited. We propose an iterative approach to estimate DOAs and grid offset jointly. We first represent the off-grid DOAs by the sum of two items, i.e., the presumed grid and its unknown grid offset. The offset is the distance from the true DOA to the neighbor grid point, which lies in a bounded interval. By introducing a convex majorization function to enforce sparsity, we then reformulate the sparse recovery problem with the unknown offset vector. Finally we iteratively decrease the majorization function to amend the offset vector such that the off-grid DOAs can be properly represented. We show by numerical simulations that the proposed method outperforms SS-MUSIC [6] and all on-grid based LASSO method [11] in terms of detection ability, resolution ability, and estimation accuracy.

The rest of the paper is organized as follows. Section 2 introduces the system model and the difference coarray with a larger aperture. Section 3 formulates the off-grid problem and presents the proposed sparse DOA estimation method. The estimation performance is evaluated in Section 4. Section 5 concludes this paper.

The following notations are used throughout this paper. Matrices (vectors) are represented by upper-case (lower-case) bold characters. In particular, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the complex conjugation, transpose and conjugate transpose of a matrix, respectively. $\mathrm{vec}(\cdot)$, $\mathrm{E}(\cdot)$, $\otimes$ and $\odot$ are operators for vectorization, expectation, Kronecker product, and Hadamard product, respectively. $\|\cdot\|_0$, $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively denote l_0, l_1, and l_2 norm. $\mathrm{real}(\cdot)$ and $\mathrm{imag}(\cdot)$ represent the real and imaginary part operations. $\mathcal{CN}(\mathbf{a}, \mathbf{B})$ denotes complex Gaussian distribution with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{B}$.

2. Signal model and preliminaries

Consider a co-prime sensor array which can be decomposed into two uniform linear subarrays. The first subarray consists of $2M$ sensors with inter-element spacing $N\lambda/2$, whereas the second consists of N sensors with inter-element spacing $M\lambda/2$. Here, M and N are co-prime integers. The array sensors are located at

$$\mathbb{L} = \{mN\lambda/2 | 0 \le m \le 2M-1\} \cup \{nM\lambda/2 | 0 \le n \le N-1\}. \tag{1}$$

Since the first sensor of the two subarrays is collocated, the total number of sensors in the co-prime array is $2M + N - 1$.

Assume K narrowband uncorrelated sources from directions $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]$ impinging on the array simultaneously. Therefore, the signal received by the array at time $t\,(1 \le t \le T)$ can be expressed as

$$\begin{aligned}\mathbf{x}(t) &= \sum_{k=1}^{K} \mathbf{a}(\theta_k)\, s_k(t) + \mathbf{n}(t) \\ &= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t).\end{aligned} \tag{2}$$

Here $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{(2M+N-1)\times K}$ is the array manifold matrix, where $\mathbf{a}(\theta_k)$ is the $(2M+N-1) \times 1$ steering vector for source k with its ith element taken as $e^{j\frac{2\pi}{\lambda} l_i \sin\theta_k}$, $l_i \in \mathbb{L}$. $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ is the source signal vector with $s_k(t)$ distributed as $\mathcal{CN}(0, \sigma_k^2)$. The elements of noise vector $\mathbf{n}(t)$ are assumed to be independent and identically distributed (i.i.d.) random variables following the complex Gaussian distribution $\mathcal{CN}(0, \sigma^2 \mathbf{I}_{2M+N-1})$. T is the number of snapshots.

The covariance matrix of data vector $\mathbf{x}(t)$ is obtained as

$$\begin{aligned}\mathbf{R_{xx}} &= E\left[\mathbf{x}(t)\mathbf{x}^H(t)\right] = \mathbf{A}\mathbf{R_{ss}}\mathbf{A}^H + \sigma^2 \mathbf{I}_{2M+N-1} \\ &= \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k) + \sigma^2 \mathbf{I}_{2M+N-1},\end{aligned} \tag{3}$$

where $\mathbf{R_{ss}} = E\left[\mathbf{s}(t)\mathbf{s}^H(t)\right] = diag\left(\left[\sigma_1^2, \sigma_2^2, \cdots, \sigma_K^2\right]\right)$ is the source covariance matrix. In practice, the covariance matrix $\mathbf{R_{xx}}$ is estimated by using the available T samples, i.e.,

$$\widehat{\mathbf{R}}_{\mathbf{xx}} = \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}(t)\mathbf{x}^H(t). \tag{4}$$

By vectorizing the covariance matrix $\mathbf{R_{xx}}$, we have

$$\mathbf{z} = \mathrm{vec}(\mathbf{R_{xx}}) = \boldsymbol{\Phi}(\theta_1, \theta_2, \cdots, \theta_K)\,\mathbf{p} + \sigma^2 \mathbf{1}_n, \tag{5}$$

where $\mathbf{p} = \left[\sigma_1^2, \cdots, \sigma_K^2\right]$, $\boldsymbol{\Phi}(\theta_1, \theta_2, \cdots, \theta_K) = \left[\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \cdots, \mathbf{a}^*(\theta_K) \otimes \mathbf{a}(\theta_K)\right]$, and $\mathbf{1}_n = \mathrm{vec}(\mathbf{I}_{2M+N-1}) = \left[\mathbf{e}_1^T, \mathbf{e}_2^T, \cdots, \mathbf{e}_{2M+N-1}^T\right]^T$ with $\mathbf{e}_i$ denoting a vector with all zero elements, except for the ith term being one. From (2) and (5), the vector $\mathbf{z}$ amounts to the received data from a coherent signal sources vector $\mathbf{p}$ with a single snapshot, and $\sigma^2 \mathbf{1}_n$ behaves like a deterministic noise term. The distinct columns in $\boldsymbol{\Phi}$ can be regarded as the steering vector of a larger virtual array which has sensors located at $l_i - l_j$, with $l_i, l_j \in \mathbb{L}$ and $1 \le i, j \le 2M + N - 1$. The resulting coarray has an extended aperture, which can be exploited to increase the DOFs and the detect ability thereby.

3. Sparse direction of arrival estimation with off-grid targets

3.1. Sparse representation

In order to estimate DOAs from (5), SS-MUSIC in [6] is feasible, however, it requires the source number K *a priori*. To tackle with this problem, sparsity based methods have been proposed in [11,22]. Specifically, a basis that grids spatial domain is usually required, i.e., one should divide the range of interest into some