Digital Signal Processing 62 (2017) 11–25

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Transfer orthogonal sparsifying transform learning for phase retrieval

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ARTICLE INFO

Article history:
Available online 27 October 2016

Keywords:
Sparsifying transform learning
Phase retrieval
Image processing
Transfer learning

ABSTRACT

The phase retrieval (PR) problem of recovering an image from its Fourier magnitudes is an important issue. Several PR algorithms have been proposed to address this problem. Recent efforts of exploiting sparsity were developed to improve the performance of PR algorithms, such as the reconstruction quality, robustness to noise, and convergence behavior. In this paper, we propose a novel sparsity-based algorithm, which can adaptively learn an orthogonal sparsifying transform, and reconstruct the image simultaneously from the Fourier magnitudes. However, the estimated images at the early iterations are extremely bad. Training samples from these images cannot provide much useful information for sparsifying transform learning. To avoid unnecessary updating, an orthogonal sparsifying transform learning method based on transfer learning is proposed. Through transfer learning, we transfer the fixed sparsifying transform to an adaptive one. We apply this new sparsifying transform learning method to PR, and exploit the alternating directions method of multipliers (ADMM) technique to solve the formulated problem. Since the learnt sparsifying transform is adaptive to data, it favors better sparsity. Using this learnt sparsifying transform for image reconstruction can improve the reconstruction quality at low oversampling ratios. Experimental results show that the proposed PR algorithm can improve nearly 6 dB compared with the recently proposed PR-TIHP-l_1 algorithm in terms of the average PSNR (Peak Signal to Noise Ratio) at oversampling ratio 2.47, 2.53, 2.59. Moreover, our algorithm is robust to noise and has better convergence behavior heuristically.

1. Introduction

Fourier phase retrieval (PR), namely recovering the signal from Fourier magnitudes, is studied by researchers in various disciplines, such as crystallography [1], astronomy [2], optical imaging [3,4], signal processing [5–7] and so on. In these fields, it is difficult to design sophisticated measuring setups to allow direct recording of the phase, which results in the lack of phase information. However, the phase encodes critical structural information about the signal [3]. Interestingly, an alternative called algorithmic phase retrieval is arising in these fields. In general, the PR problem is ill-posed, therefore, priors on the signal are needed to enable its recovery. Several priors, such as non-negativity, support information and sparsity, have been exploited [3,4]. According to solvers and priors, the existing PR methods can be roughly categorized into the following classes: alternating projection methods [8–12], sparsity-based alternating projection methods [13–16], nonlinear compressed sensing methods [17–19], convex optimization methods [20–22] and regularized methods [5–7].

The core idea of alternating projection methods is to start with an initial guess and project the estimate onto the Fourier magnitude constraint set and the object constraint set alternately until the terminated condition is reached. The Fourier magnitude constraint set guarantees that the spectrum of the reconstructed image has the same Fourier magnitude as the measurement. In general, researchers often focus on the object constraint set, which can utilize different priors for reconstruction. Hence, various PR algorithms with different priors were developed. The so-called G–S algorithm [8] was proposed in the seminal work of alternating projection methods pioneered by Gerchberg and Saxton in 1972, which dealt with the PR problem of complex images. The object constraint set in G–S algorithm guarantees that the intensities of the complex image are consistent with the measured data. However, the G–S algorithm often falls into the stagnation, thus, usually yields the local optimized solution. To address this issue, Fienup [9] suggested a PR algorithm called HIO (Hybrid Input–Output) algorithm that incorporates the feedback mechanism and the support constraint into PR. Subsequently, various alternating projection methods [10–12], such as RAAR (Relaxed Averaged Alternating Reflection) algorithm [10], difference map algorithm [11],

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http://dx.doi.org/10.1016/j.dsp.2016.10.014
1051-2004/

were developed. In fact, these methods can be regarded as the modification or extension of the HIO algorithm or the GS algorithm. Recently, inspired by advances of CS (Compressed Sensing) [23], the sparsity has been explored by various researchers for PR. The sparsity-based alternating projection methods incorporate sparsity priors into object constraint sets. Theoretically, any alternating projection methods can combine the sparsity with other priors to improve the performance, examples like the Max-K algorithm [13], the RAAR algorithm of utilizing the Shearlet sparsity [14], the compressed HIO algorithm [15], etc. Experimental results [13–15] indicate that the sparsity-based alternating projection methods outperform the traditional alternating projection methods in terms of reconstruction quality or convergence speed.

Another class of PR algorithms is the nonlinear compressed sensing (NLCS) method [17–19], which solves the PR problem within the NLCS framework. The NLCS deals with the problem of recovering the signal from nonlinear measurements. Since the PR problem is a special NLCS problem, any NLCS method can be used to solve the PR problem theoretically. Ohlsson et al. [18] proposed the nonlinear basis pursuit algorithm that solves the NLCS problem using the convex relaxation technique. Blumensath et al. [19] approximated the nonlinear sampling operator by using an affine Taylor series, and solved the resulted problem by the iterative hard thresholding (IHT) algorithm. Differing from the spirits of the NLCS, convex optimization methods often translate the corresponding non-convex problems into convex ones. Example like the PR algorithm based on phase lift technique [20,21], they exploit the phase lift technique to formulate a low rank PR optimization problem, and translate the resulted non-convex problem to a convex one. The resulted problem is solved by using the convex optimization technique. However, the algorithm using phase lift is not suitable for large-scale problems.

The regularized PR methods formulate the optimization problem via combining the data fidelity term with a fine designed regularization term. Here, the data fidelity term indicates the signal is consistent with the measurement and the regularization term characterizes the prior. Yang et al. [5] incorporated the l_1 regularization for PR, and proposed the alternating directions method of multipliers (ADMM) [24] technique to solve the resulted problem. However, their algorithm cannot recover signals that are non-sparse in spatial domain. To cope with this issue, Shi et al. [6] extended this framework, and exploited the sparsity in the Translation Invariant Haar Pyramid (TIHP) tight frame for PR. Additionally, the DOLPHIn (DictiOnary Learning for PHase retrIeval) algorithm [7] is proposed by using adaptive sparse representation. This algorithm addresses the PR problem of utilizing the coded diffraction pattern (CDP) sampling model. Readers who are interested in the CDP model can refer to the papers [25–27].

In this paper, we mainly consider the task of recovering an image from far-field data. Although the aforementioned methods have been proposed to address this problem, most of them are either suffering from failing to reconstruct high quality images at low oversampling ratios, or being sensitive to noise. Recent efforts are often exploiting analytical sparsifying transform, such as shearlet transform and wavelet transform, for improving the reconstruction quality. However, since the sparsifying transform may not always be optimal to each image, these sparsity-based algorithms fail to reconstruct high quality images when the oversampling ratios are low. To address these issues, we propose a novel PR framework based on an adaptive sparsifying transform via utilizing a transform learning method and transfer learning. Our main innovations have two aspects: (1) an efficient sparsifying transform learning method based on transfer learning is proposed; (2) a PR framework based on the proposed sparsifying transform learning method is proposed. Since learning process is time-consuming, we incorporate the orthogonal structure of the learnt sparsifying transform into the learning process to reduce the learning time. The bad estimated images at early stages of iterations contain most noise-like components, which carry little useful information for training. The sparsifying transform learning from this type of estimated images cannot capture complicated structures or details of an image. Consequently, reconstructing an image using this learnt sparsifying transform often yields a bad reconstruction. Keep this fact in mind, we propose a transfer transform learning via transfer learning. In fact, Chen et al. [28] have proposed a transfer overcomplete dictionary learning method via domain adaptation. Their algorithm can balance the tradeoff between dictionary learning speed and accuracy by transferring an existing dictionary to an adaptive one. Motivated by this method, we propose an efficient Transfer Orthogonal sparsifying Transform Learning (TOTL) method based on the recently proposed sparsifying transform learning model, which is different from the synthesis sparse model in [28]. We formulate a regularized PR problem via the proposed TOTL method, and then propose an efficient method to solve the corresponding problem. Numerical simulations validate the effectiveness of our algorithm.

2. Related work and our main work

2.1. Phase retrieval algorithms

The Fourier PR sampling model can be described as

$$\mathbf{b} = |\mathbf{F}\mathbf{x}| + \mathbf{n} \tag{1}$$

where $|\cdot|$ denotes the magnitude operator, $\mathbf{F}$ is the Fourier transform matrix, $\mathbf{x}$ represents the underlying image, $\mathbf{b}$ denotes its corresponding measurement, and $\mathbf{n}$ represents the noise vector. PR problem is about recovering the image $\mathbf{x}$ from the nonlinear measurement $\mathbf{b}$. Mathematically, the problem can be formulated as a feasible problem

$$\text{find} \quad \mathbf{x} \in \mathrm{M} \cap \mathrm{S}, \tag{2}$$

here $\mathrm{M} = \{\mathbf{x} \in \mathbb{R}^N \mid |\mathbf{F}\mathbf{x}| = \mathbf{b}\}$ represents the Fourier magnitude constraint set and S denotes the object constraint set, such as non-negativity, support constraint set or sparsity constraint set. For support constraint, S is defined as $\mathrm{S} = \{\mathbf{x}(r) \mid \mathbf{x}(r) \neq 0 \text{ for some } r \in \Omega \text{ and } \mathbf{x}(r) = 0 \text{ for } r \notin \Omega\}$, which indicates the set of signals have the non-zero support in Ω (the index set of the support). The alternating projection methods solve the above problem via projecting the estimated signal onto the two constraint sets alternatively. The sparsity-based alternating projection methods often incorporate the sparsity prior into the object constraint set to improve reconstruction quality or speed up convergence. Mukherjee et al. [13] proposed the so-called Max-K algorithm that can be regarded as solving a feasible PR problem, where the goal is to find a signal satisfying both the sparsity constraint set and the Fourier magnitude constraint set. They enforced sparsity via solving a sparse coding problem, and retained the K largest non-zero coefficients in the thresholding process, thus termed Max-K; Loock et al. [14] incorporated the sparsity in a Shearlet frame into the RAAR algorithm and exploited l_1 norm to promote sparsity; Qin et al. [15] incorporated the spatial sparsity of the image into the HIO algorithm, and combined the phase diversity to recover the image; Gaur et al. [16] exploited the total variation (TV) operator to incorporate the sparsity in gradient domain into the HIO algorithm; in the object constraint, Gaur et al. performed the TV reduction procedure by using a gradient descent process. Differing from the PR feasible problem, the PR problem can also be formulated as a regularized problem that can be solved by optimization theory. The alternating projection methods often incorporate image inherent priors into the object constraint set, while the regularized PR