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Performance analysis of the deficient length NSAF algorithm and a variable step size method for improving its performance

Yi Yu [a,b], Haiquan Zhao [a,b,*]

[a] Key Laboratory of Magnetic Suspension Technology and Maglev Vehicle, Ministry of Education, Chengdu, 610031, China
[b] School of Electrical Engineering, Southwest Jiaotong University, Chengdu, 610031, China

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ABSTRACT

In all presented analyses of the normalized subband adaptive filter (NSAF) algorithm, there is a common assumption that the length of the adaptive filter is equal to that of the unknown system. In many practices, however, the adaptive filter usually works in an under-modeling situation. Namely, the length of the adaptive filter is less than that of the unknown system. Therefore, for this case, the existing analysis results for the NSAF algorithm are not applicable. In this paper, we analyze the performance of the deficient length NSAF algorithm based on some reasonable assumptions and approximations. More precisely, the expressions that characterize the transient-state and steady-state mean-square behavior of the algorithm are presented. Simulation results in various scenarios support our theoretical expressions. In addition, based on our analyses, a variable step size NSAF algorithm suitable for the under-modeling case is developed, which improves the performance.

1. Introduction

As a subfield of modern signal processing theory, adaptive filtering algorithms play a very important role in some practical applications such as system identification, active noise control, beamforming, channel equalization, and echo cancellation [1–3]. Among the existing algorithms, the normalized least mean square (NLMS) is well-known, due to its simplicity and robustness to the power of input signal. Furthermore, to obtain both fast convergence rate and low steady-state error, many variable step size NLMS algorithms were proposed [4–6]. Nevertheless, these algorithms will suffer from slow convergence when the input signals are colored, also called the correlated signals.

To deal with this problem, in a recent decade, the subband adaptive filter (SAF) has received significant attention. In subband adaptive filtering, the colored input signal is divided into almost mutually exclusive multiple subband signals and then critically decimated so that each decimated subband signal is approximately white, thereby improving the convergence performance [3]. In contrast to the conventional subband structure, the multiband structure of SAF has no aliasing and band edge effects, and thus it is more effective [3,7]. Based on this multiband-structure, Lee and Gan proposed the normalized SAF (NSAF) algorithm by means of the principle of least perturbation [7]. This algorithm exhibits faster convergence rate than the NLMS algorithm for the colored input signals, due mainly to the fact that it inherits the decorrelating property of SAF [8]. Apart from the above property, the NSAF algorithm has almost the same computational complexity as the NLMS algorithm for applications of long adaptive filter. As a matter of fact, the NSAF algorithm will reduce to the NLMS algorithm when number of subbands is one. Following this algorithm, to balance a tradeoff of the NSAF algorithm between the fast convergence rate and low final estimation error which is controlled by the fixed step size and/or regularization constant, several variable step size [9–12] and/or variable regularization constant [13–15] versions were developed successively.

The performance analysis is a very important research topic for adaptive filtering algorithm, because it is very beneficial to predict the behavior of a specific adaptive filtering algorithm and to provide some guidelines to further improve the filter performance [16–26]. In [16] and [17], the steady-state mean-square error (MSE) results of the NSAF algorithm for the fixed step size and regularization constant were analyzed, respectively. However, in system identification and channel estimation scenarios, analyzing the mean-square-deviation (MSD) of the algorithm is more reasonable than the MSE of that, because the goal in these applications is to identify the system impulse response. In [18], Yin and Mehr studied the MSD behavior (including the transient-state and steady-state) of the NSAF algorithm based on the paraunitary

* Corresponding author at: Key Laboratory of Magnetic Suspension Technology and Maglev Vehicle, Ministry of Education, Chengdu, 610031, China.
E-mail addresses: yuyi_xyuan@163.com (Y. Yu), hqzhao_swjtu@126.com (H. Zhao).

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assumption of the analysis filter bank. Moreover, in this analysis, three methods were used to solve some hyperelliptic integrals, i.e., the Lobatto quadrature, chi-square, and partial fraction methods, at the expense of high computational complexity especially for a long filter. The previous theoretical analyses in [16–18] obtain good agreement with the simulated results, but they require the assumption that the length of the adaptive filter is equal to that of the unknown impulse response. However, in many practical applications such as acoustic echo cancellation [4,5], the length of unknown impulse response is unknown and very large, which would yield under-modeling situation. Under-modeling here means that, the length of the adaptive filter is less than that of the unknown impulse response, which is also called the deficient length. Consequently, the previous theoretical results on the sufficient length NSAF algorithm do not necessarily apply to the deficient length situation. For such scenarios, the performance of many algorithms has been studied in the literature such as the LMS [19,20], the frequency-domain block LMS (FBLMS) [21], and the distributed LMS [22,23]. To the best of our knowledge, however, there are no available studies to accurately evaluate the performance of the deficient length NSAF algorithm. To this end, benefited from [19–23], we will focus on analyzing the performance of the deficient length NSAF algorithm under some reasonable assumptions and approximations, involving the transient-state and steady-state behaviors. Following the previous analysis processes, we also derive a variable step size NSAF algorithm which provides good performance in under-modeling situation. The proposed theoretical results and algorithm are also supported by extensive simulations.

The remainder of this paper is organized as follows. In the next section, we briefly review the NSAF algorithm. In Section 3, the performance of the deficient length NSAF algorithm is analyzed in mean and mean-square senses. In Section 4, simulations are performed to verify the theoretical analysis. Section 5 develops the variable step size NSAF algorithm for under-modeling. Finally, conclusions are presented in Section 6.

Notation: for the ease of reference, the main notations used in this paper are as follows: $(\cdot)^T$ denotes the transpose of a vector or matrix; $E\{\cdot\}$ denotes the mathematical expectation of a random variable; $\|\cdot\|$ stands for the l_2-norm of a vector; $\mathbf{0}_{M\times L}$ is a $M\times L$ zero matrix; and $\mathbf{I}_L$ is the identity matrix of size $L\times L$.

2. The standard NSAF algorithm

Consider the desired signal $d(n)$ that arises from the model

$$d(n)=\mathbf{u}^T_{L\mathrm{opt}}(n)\mathbf{w}_{L\mathrm{opt}}+\eta(n), \tag{1}$$

where $\mathbf{w}_{L\mathrm{opt}}=[w_{1,o},w_{2,o},\ldots,w_{L,o}]^T$ is the unknown L-dimensional vector to be estimated with an adaptive filter, $\mathbf{u}_{L\mathrm{opt}}(n)=[u(n),u(n-1),\ldots,u(n-L+1)]^T$ is the input signal vector, and $\eta(n)$ is the system noise.

Fig. 1 shows the multiband-structure diagram of the SAF [7] which has been used to derive the NSAF family, where N denotes number of subbands. The desired signal $d(n)$ and input data $u(n)$ are partitioned into multiple subband signals $d_i(n)$ and $u_i(n)$ through the analysis filter bank $\{H_i(z),\ i=0,1,\ldots,N-1\}$, respectively. The subband signals $y_{i,D}(k)$ and $d_{i,D}(k)$ are obtained by critically decimating $y_i(n)$ and $d_i(n)$. Here, n and k are used to indicate the original sequences and the decimated sequences, respectively. It is easy to know that the subband error signals $e_{i,D}(k)$ for $i=0,1,\ldots,N-1$ are expressed as

$$\begin{aligned}e_{i,D}(k)&=d_{i,D}(k)-y_{i,D}(k)\\&=d_{i,D}(k)-\mathbf{u}_i^T(k)\mathbf{w}(k)\end{aligned} \tag{2}$$

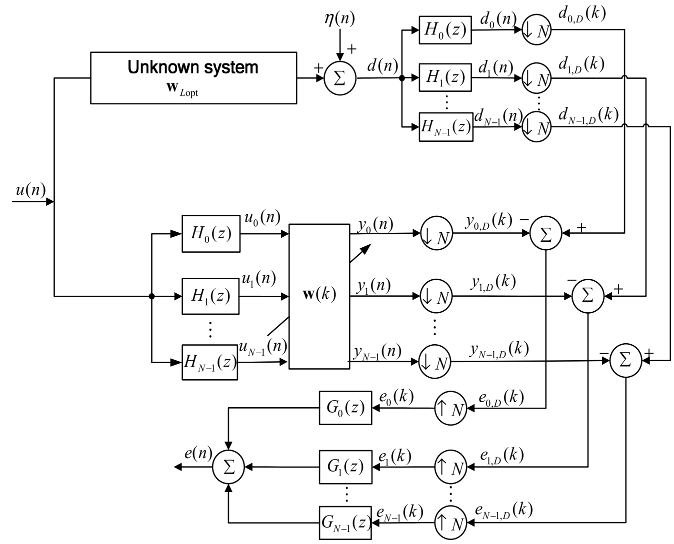

Fig. 1. Multiband-structure diagram of SAF.

where $\mathbf{w}(k)=[w_1(k),w_2(k),\ldots,w_M(k)]^T$ denotes the tap-weight vector of the adaptive filter with length M, $\mathbf{u}_i(k)=[u_i(kN),u_i(kN-1),\ldots,u_i(kN-M+1)]^T$, and $d_{i,D}(k)=d_i(kN)$.

In [7], the standard NSAF algorithm for updating the tap-weight vector is expressed as

$$\mathbf{w}(k+1)=\mathbf{w}(k)+\mu\sum_{i=0}^{N-1}\frac{e_{i,D}(k)\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2} \tag{3}$$

where μ is the step-size. In practical applications, the length of the unknown system might be very large, which would result in under-modeling situation, i.e., $M<L$ [4,5]. In this case, therefore, we will study the performance of the NSAF algorithm.

3. Performance analyses

3.1. Some assumptions and preliminaries

For convenience of analysis, the following assumptions are necessary.

Assumption 1. The system additive noise $\eta(n)$ is a white Gaussian process with zero-mean and variance σ^2_η.

Assumption 2. The cosine modulated analysis filter bank for partitioning the input signal $u(n)$ and the observed output $d(n)$ is assumed to be paraunitary [16,18,24,26]. Based on this assumption, we have

$$d_{i,D}(k)=\mathbf{u}^T_{L\mathrm{opt},i}(k)\mathbf{w}_{L\mathrm{opt}}+\eta_i(k) \tag{4}$$

where the subband noises $\eta_i(k)$ for $i=0,1,\ldots,N-1$ are also zero-mean white but variances $\sigma^2_{\eta_i}=\sigma^2_\eta/N$ which is obtained from $\eta(n)$ through the analysis filter bank.

Assumption 3. The tap-weight vector $\mathbf{w}(k)$, the subband input vector $\mathbf{u}_i(k)$, and the subband noise $\eta_i(k)$ are statistically independent. This type of assumption is commonly done in the context of adaptive filtering algorithms, see [1,16–26].

Assumption 4. Due to the inherent decorrelating property of SAF, each decimated subband input signal can be assumed to be white, i.e., $E\{\mathbf{u}_i(k)\mathbf{u}_i^T(k)\}\approx\mathbf{I}_M\sigma^2_{u_i}$ and $E\{\|\mathbf{u}_i(k)\|^2\}\approx M\sigma^2_{u_i}$ [17,24], where