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DOA estimation based on multi-resolution difference co-array perspective ☆

Jianyan Liu [a,b], Yanmei Zhang [a], Yilong Lu [b], Weijiang Wang [a,*]

[a] School of Information and Electronics, Beijing Institute of Technology, Beijing 100081, PR China
[b] School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore

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ABSTRACT

This paper presents two kinds of K-level co-prime linear array geometries and the corresponding direction of arrival estimation algorithm based on the multi-resolution difference co-array (MRDCA) perspective. The MRDCA can simultaneously improve the degree of freedom and the angle-resolution by utilizing a class of virtual sparse uniform linear arrays generated by vectorizing the covariance matrix of the received observations of the K-level large scale sparse array. Compared to the prior two level co-prime/nested arrays, the aperture and the angle-resolution can be significantly increased with Kth power law for the K-level array, while the dimension of its scanning space is reduced to $1/K$ resulted from the spatial aliasing of the MRDCA. As a result, a low-complexity DOA estimation algorithm is proposed by combining a multi-resolution estimation at each level of sparse MRDCA and a followed probability decision strategy which aims at effectively identifying the genuine DOAs and excluding the replicas. In the end, the simulation results are provided to numerically validate the performance of the proposed array geometries.

1. Introduction

Direction of arrival (DOA) estimation is a critical problem in array signal processing. In the past few decades, many high-resolution subspace fitting approaches (e.g. MUSIC [1], ESPRIT [2] and their variants) have been proposed mainly based on overdetermined model (more sensors than sources) and uniform linear arrays (ULAs) with spacing of around half a wavelength to avoid spatial aliasing. In such a situation, to achieve very high angle-resolution and/or detect many sources with a ULA, it requires a very large aperture with massive antenna elements, which leads to formidable system complexity and costs. Therefore, some non-uniform linear arrays (NLAs) have been introduced into array signal processing field, dating back to minimum-redundancy arrays (MRAs) [3]. By directly constructing an augmented covariance matrix [4,5] or indirectly utilizing the transformation of the augmented matrix into a positive definite Toeplitz matrix [6], such MRAs allow adjacent physical element spacing to be greater than half a wavelength and also provide a solution to detect more sources than sensors. Recently, the introduction of nested array in [7] and co-prime array in [8] (their variants: super nested array [9,10], generalized co-prime array [11] and fourth-order based co-prime array [12]) has created renewed interest in NLAs. A new difference co-array (DCA) perspective for underdetermined array signal model is provided in [7] by directly vectorizing the covariance matrix of the measurements from such NLAs. This vectorized signal can be viewed as an output from a virtual array whose element positions are defined by difference positions of physical sensors. Therefore, many array signal processing algorithms, such as DOA estimation and beam-forming, can be performed based on such virtual co-arrays instead of physical arrays. As a result, the corresponding degrees of freedom (DOFs) (enhanced from $O(N)$ to $O(N^2)$) allow to find more sources than sensors [13]. Furthermore, multi-frequency techniques have been proposed in [14] to fill the missing co-array elements, thereby enabling the co-prime array to effectively utilize all of the offered DOFs.

After obtaining the increased DOFs from these NLAs, DOA estimation can be performed mainly based on two main methodologies: the subspace fitting and the sparsity recovery. For the former, a rank restoring method, such as the spatial smoothing [15,16] and the spatial smoothed matrix direct constructing [17], should be implemented prior to the application of MUSIC algorithm since the equivalent input sources are fully coherent. For the latter, DOAs are recovered by utilizing the sparsity property of the sources in

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* Corresponding author.
E-mail addresses: N1509412L@e.ntu.edu.sg (J. Liu), zhangym0726@163.com (Y. Zhang), eylu@ntu.edu.sg (Y. Lu), wangweijiang@bit.edu.cn (W. Wang).

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the spatial domain [18–20], where the problem of off-grid sources was discussed in [21,22], coherent sources in [23] and wide-band sources in [24].

Inspired by these methods and NLA geometries, two kinds of multi-level co-prime array geometries are proposed in this article for DOA estimation applications with a better angle-resolution. The prior co-prime array is mainly based on two-level geometry to generate a set of large consecutive lags in its virtual DCA (since the K-level geometry only obtains a slight improvement over the two level one). Then, the enhancement of corresponding virtual aperture obeys the second power law (from $O(N)$ to $O(N^2)$). Our proposed K-level geometries are more flexible consisting of more than two levels (i.e. K-level), therefore the enhancement of virtual aperture obeys the Kth power law (from $O(N)$ to $O(N^K)$). Obviously, the K-level geometries can provide a more efficient way for improving the aperture as well as the angle-resolution. By vectorizing the covariance matrix of measurements from the K-level array, a class of virtual sparse ULAs with different angle-resolutions and different inter-element spacing are generated (called multi-resolution difference co-arrays). Then a class of DOA estimation with different angle-resolutions and scanning spaces can be performed based on such MRDCAs, thereby enabling the effective utilization of both the offered DOFs and the increased aperture. In a sense, the multi-resolution concept had been introduced to digital beam-forming with high angle-resolution in our prior work [25,26]. Here for DOA estimation object, one need to find the genuine DOAs and exclude the replica ones, therefore, a novel probability decision strategy is proposed by combining the results of multi-level DOA estimation.

The article is organized as follows. In Section 2, we review the virtual array signal model and establish some important theorems and properties about the angle-resolution of sparse ULA. In Section 3, the formation of two kinds of K-level co-prime arrays and their corresponding configurations of MRDCAs are derived. Section 4 presents the multi-resolution DOA estimation algorithm and the followed probability decision strategy. In Section 5, the simulation results are provided to numerically validate the method. Section 6 concludes this paper.

The following notations are used throughout: We use lower-case bold characters to denote vectors (e.g. $\mathbf{a}$), upper-case bold for matrices (e.g. $\mathbf{A}$) and upper-case outline for a set (e.g. $\mathbb{A}$). For a matrix $\mathbf{A}$, the symbols $\mathbf{A}^*$, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the conjugation, transpose and conjugate transpose, respectively. $diag\{\mathbf{A}\}$ denotes a column vector consisting of the main diagonal elements of matrix $\mathbf{A}$. $diag\{\mathbf{a}\}$ denotes a diagonal matrix that uses the elements of vector $\mathbf{a}$ as its diagonal elements. $vec\{\mathbf{A}\}$ denotes vectorization, which converts the matrix $\mathbf{A}$ into a column vector by stacking the columns of the matrix $\mathbf{A}$ on top of one another. The symbols $\odot$ and $\otimes$ respectively denote the Khatri–Rao product and Kronecker product between two matrices of appropriate size, respectively. The symbol $\mathbb{Z}$ denotes the set of all integers. The symbol $\mathbb{Z}_a^b$ $(a \le b)$ denotes the set consisting of integers from a to b, $\mathbb{Z}_a^b \triangleq \{a, a+1, \ldots, b\}$. $gcd(\bullet)$ denotes the greatest common divisor.

2. Signal model and problem formulation

2.1. Definition

Definition 1 *(Set operation).* For any two sets of integers $\mathbb{A}$ and $\mathbb{B}$, one can define the following operations:

$$\begin{aligned}
\textbf{Intersection Set: } & \mathbb{A} \cap \mathbb{B} = \{a : \forall a \in \mathbb{A} \text{ and } a \in \mathbb{B}\} \\
\textbf{Union Set: } & \mathbb{A} \cup \mathbb{B} = \{a : \forall a \in \mathbb{A} \text{ or } a \in \mathbb{B}\} \\
\textbf{Sum Set: } & \mathbb{A} + \mathbb{B} = \{a + b : \forall a \in \mathbb{A}, \forall b \in \mathbb{B}\} \\
\textbf{Difference Set: } & \mathbb{A} - \mathbb{B} = \{a - b : \forall a \in \mathbb{A}, \forall b \in \mathbb{B}\} \\
\textbf{Translation Set: } & \lambda + \mathbb{A} = \{\lambda + a : \forall a \in \mathbb{A}, \lambda \in \mathbb{Z}\} \\
\textbf{Dilation Set: } & \lambda \mathbb{A} = \{\lambda a : \forall a \in \mathbb{A}, \lambda \in \mathbb{Z}\}
\end{aligned}$$

Definition 2 *(Difference co-array of linear arrays [7]).* Considering a linear array of M sensors located at $\mathbb{A} = \{a_1, a_2, \ldots, a_N\}d_0$, where $a_i d_0$ denotes the ith sensor position, the difference co-array of the given array is defined as the array of sensors located at $\mathbb{D}$, where $\mathbb{D} \triangleq \mathbb{A} - \mathbb{A}$.

In the prior DCA model, a virtual ULA with a minimal inter-element spacing d_0 and without holes is desirable for spatial sampling purpose. In this article, we further develop this notion into the multi-resolution difference co-array (MRDCA) perspective by considering a set of virtual ULAs with different inter-element spacing. Specifically, the MRDCA $\mathbb{R}(\alpha, \mathrm{L}_\alpha)$ is a virtual ULA contained in the difference co-array of the given array and with inter-element spacing αd_0 up to one-sided aperture $\alpha d_0 L_\alpha$. Therefore, in mathematical formulation, $\mathbb{R}(\alpha, \mathrm{L}_\alpha) \triangleq \alpha d_0 \mathbb{Z}_{-\mathrm{L}_\alpha}^{+\mathrm{L}_\alpha} \subseteq \mathbb{D}$.

2.2. Review of DOA estimation based on DCA

Considering that a non-uniform linear array with M sensors at locations $\mathbb{A} = \{l_1, l_2, \cdots, l_M\}$ in units of half a minimal wavelength in space and D far-field narrowband sources impinges to the array from directions θ_i, $i = 1, 2, \cdots, D$, therefore the array output can be expressed as (1):

$$\mathbf{x}(n) = \sum_{k=1}^{D} \mathbf{a}(\theta_k) s_k(n) + \mathbf{w}(n) = \mathbf{A}(\theta) s(n) + \mathbf{w}(n) \tag{1}$$

where $\mathbf{w}(n)$ denotes zero-mean and statistically independent additive Gaussian white noise. $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_D)]$ is the $M \times D$ array manifold matrix, whose kth column is the steering vector $\mathbf{a}(\theta_k)$ corresponding to the kth DOA. The mth element of $\mathbf{a}(\theta_k)$ is defined as $\mathbf{a}_m(\theta_k)$ (where $\mathbf{a}_m(\theta_k) = e^{j2\pi l_m v(\theta_k)}$ and $v(\theta_k) = \frac{d_0}{\lambda} \sin(\theta_k)$). Then the covariance matrix can be formed by (2):

$$\mathbf{R}_{xx} = E\left[\mathbf{x}\mathbf{x}^H\right] = \mathbf{A}(\theta)\mathbf{R}_{ss}\mathbf{A}(\theta)^H + \sigma_n^2 \mathbf{I}_M \tag{2}$$

Based on difference co-array perspective [7], the covariance matrix in (2) can be directly vectorized into an $M^2 \times 1$ vector:

$$\begin{aligned}
\mathbf{z} &= vec(\mathbf{R}_{xx}) \in \mathbb{C}^{M^2 \times 1} \\
&= (\mathbf{A}^*(\theta) \odot \mathbf{A}(\theta))\mathbf{p} + \sigma_n^2 \, vec\{\mathbf{I}_M\} \\
&= \hat{\mathbf{A}}(\theta)\mathbf{p} + \sigma_n^2 \mathbf{l}_M
\end{aligned} \tag{3}$$

where $\mathbf{p} = diag\{\mathbf{R}_{ss}\}$ and $\mathbf{l}_M = vec\{\mathbf{I}_M\}$.

By comparing (3) and (1), one can find that $\mathbf{z}$ behaves like a single-snapshot output signal from an array with sensors at locations $(\mathbb{A} - \mathbb{A})$. $\mathbf{p}$ and $\mathbf{l}_M$ are equivalent input sources and noise. $\hat{\mathbf{A}}$ is an augmented array manifold. Therefore, DOA estimation can be performed based on the virtual array model (3) rather than the physical array model (1). For this purpose, there are two main methodologies: the sparsity recovery and the subspace fitting (SSF). Here we mainly focus on SSF method (more practical and more effective for engineering purpose), it proceeds as follows. The SSF method requires consecutive virtual sensor positions, therefore, one can select a uniform linear difference co-array with a minimal inter-element spacing, denoted as $\mathbb{R}(1, L_1)$. The NLA signal model (3) can be reduced into a ULA signal model (4):

$$\mathbf{z}_1 = \hat{\mathbf{A}}_1(\theta)\mathbf{p} + \sigma_n^2 \mathbf{e}_{L_1} \in \mathbb{C}^{(2L_1+1)\times 1} \tag{4}$$

where $\mathbf{e}_{L_1} = \left[\mathbf{0}_{1\times L_1} \;\; 1 \;\; \mathbf{0}_{1\times L_1}\right]^T$. Compared to the conventional physical array model, two points should be specially noted: firstly, there is only one non-zero element in $\mathbf{e}_{L_1}$, which indicates that the noise only affects the zero-lag virtual sensor. The corresponding influence can be canceled out by applying a noise power suppression to the output of the zero-lag sensor or by adopting a general algorithm for DOA estimation in the presence of non-uniform