Digital Signal Processing 62 (2017) 197–210

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Tensor-based methods for blind spatial signature estimation under arbitrary and unknown source covariance structure ☆

Paulo R.B. Gomes [a], André L.F. de Almeida [a,*], João Paulo C.L. da Costa [b], Giovanni Del Galdo [b]

[a] Department of Teleinformatics Engineering, Federal University of Ceará, Fortaleza, Brazil
[b] Institute for Information Technology, Ilmenau University of Technology, Ilmenau, Germany

ARTICLE INFO

Article history:
Available online 13 December 2016

Keywords:
Array processing
Spatial signature estimation
Tensor decomposition
Alternating least squares

ABSTRACT

Spatial signature estimation is a problem encountered in several applications in signal processing such as mobile communications, sonar, radar, astronomy and seismology. In this paper, we propose higher-order tensor methods to solve the blind spatial signature estimation problem using planar arrays. By assuming that sources' powers vary between successive time blocks, we recast the spatial and spatiotemporal covariance models for the received data as third-order PARATUCK2 and fourth-order Tucker4 tensor decompositions, respectively. Firstly, by exploiting the multilinear algebraic structure of the proposed tensor models, new iterative algorithms are formulated to blindly estimate the spatial signatures. Secondly, in order to achieve a better spatial resolution, we propose an expanded form of spatial smoothing that returns extra spatial dimensions in comparison with the traditional approaches. Additionally, by exploiting the higher-order structure of the resulting expanded tensor model, a multilinear noise reduction preprocessing step is proposed via higher-order singular value decomposition. We show that the increase on the tensor order provides a more efficient denoising, and consequently a better performance compared to existing spatial smoothing techniques. Finally, a solution based on a multi-stage Khatri–Rao factorization procedure is incorporated as the final stage of our proposed estimators. Our results demonstrate that the proposed tensor methods yield more accurate spatial signature estimates than competing approaches while operating in a challenging scenario where the source covariance structure is unknown and arbitrary (non-diagonal), which is actually the case when sample covariances are computed from a limited number of snapshots.

1. Introduction

The great interest on the use of antenna arrays in communication systems is directly related to capacity and coverage gains they can provide, as well as the possibility of implanting techniques that promote space division multiple access (SDMA) [2,3]. In the SDMA context, the knowledge of spatial signatures of the source signals is very important, which has motivated the development of several matrix-based methods in the last decades [4]. The existing solutions can be categorized into different ways depending on assumptions involving (i) the knowledge (or not) of pilot signals, (ii) the use of parametric or nonparametric models for the spatial signatures, (iii) the use of sources' statistical independency or cyclostationarity, to mention a few. In this context, blind methods are of particular interest, as they are more bandwidth-efficient and avoid tight user synchronization [5–8].

Blind estimation techniques have shown great potential to solve array signal processing problems such as beamforming and direction of arrival estimation [5–8]. However, most of these methods do not fully exploit the multidimensional structure of the received data, which may span several domains such as space, time, frequency and/or polarization. In particular, we can notice that space domain can be split into two signal dimensions (azimuth and elevation), while time domain can be divided into two dimensions (snapshots and frames).

In order to deal with such a multidimensional nature of the data signals, tensor decompositions have extensively been ap-

☆ The authors would like to thank the FUNCAP, CAPES and CNPq under the processes numbers 8888.030392/2013-01 and 303905/2014-0 for their financial support on this research. This work is an extension of our conference paper [1] published in ICASSP'2014.

* Corresponding author.
E-mail addresses: paulo@gtel.ufc.br (P.R.B. Gomes), andre@gtel.ufc.br (A.L.F. de Almeida), joaopaulo.dacosta@ene.unb.br (J.P.C.L. da Costa), giovanni.delgaldo@iis.fraunhofer.de (G. Del Galdo).

http://dx.doi.org/10.1016/j.dsp.2016.12.003
1051-2004/

plied in recent years in array signal processing and multiple-antenna communication problems. In the signal processing context, [8] proposes a transmission scheme based on power variations of the transmitted signals at successive time blocks where the blind spatial signature estimation problem is solved using two approaches: the first approach is based on the third-order PARAllel FACtor (PARAFAC) tensor decomposition [9,10], while the second one relies on a joint approximate diagonalization algorithm. In the seminal paper [12], a blind receiver for direct-sequence code division multiple access (DS-CDMA) systems is presented using the PARAFAC model. In [13] and [14], PARAFAC modeling is linked to the problem of multiple invariance sensor array processing (MI-SAP) and MIMO radar systems, respectively. In [15] the signals' polarization dimension is exploited in the tensor modeling of polarimetric arrays, while in [16] and [17] a closed-form PARAFAC solution and tensor-based ESPRIT algorithm are proposed for R-dimensional parameter estimation, respectively.

In the context of wireless communications, [18] proposes a unified PARAFAC-based modeling for tensor-based receivers with application to blind multiuser equalization. In [19] and [20], different tensor-based receivers are presented for solving the joint symbol and channel estimation problem in space-time-frequency (STF) MIMO communication systems. In the former, the PARAFAC decomposition is exploited to derive a closed-form solution based on the Khatri–Rao factorization. The latter proposes a semi-blind receiver based on alternating least squares that exploits a generalized PARATUCK2 model of the STF-MIMO transmission system. The key features motivating the use of tensor decompositions in the aforementioned works come from their powerful identifiability and uniqueness properties compared with matrix-based methods [10, 21]. Additionally, one can also benefit from the multidimensional structure of tensor data to perform noise rejection/prewhitening, as shown in [22].

In the particular context of spatial signature estimation in array processing, the existing tensor methods [8,11,13] are restricted to the PARAFAC decomposition. An approach of particular interest here is the PARAFAC-based method of [8], which exploits multiple data covariance matrices by imposing a block-wise stationary property on the source signals. It is worth mentioning that [8] assumes the source covariance matrices are diagonal and perfectly known at the receiver. However, these features correspond to the assumption of perfect decorrelation between source signals, while representing an asymptotic (ideal) situation where the data covariance matrix is perfectly computed/estimated from the received data samples. We are interesting in relaxing, or avoiding, such idealistic assumptions to deal with more general scenarios. Otherwise stated, this means that we should resort to more flexible tensor models to conveniently model the problem, which in turn have a direct impact on the receiver processing strategy to be used.

In this paper, we initially propose tensor-based methods to solve the problem of blind spatial signature estimation using planar arrays. Our methods discard idealizing assumptions about the source covariance structure, as opposed to the competing methods referred previously. Moreover, the proposed methods do not require the use of training sequences nor the knowledge about the propagation channel. By assuming that sources' transmit powers vary between successive time blocks, we recast the spatial and spatiotemporal covariance models for the received data as a third-order PARATUCK2 and fourth-order Tucker4 tensor decompositions, respectively. For each proposed method, our second contribution consists in developing iterative algorithms based on alternating least squares (ALS) which are used to blindly estimate the sources' spatial signatures. The proposed tensor methods are able to operate in a challenging scenario where sources covariance structure is unknown and arbitrary (non-diagonal), which is actually the case when sample covariances are computed from a limited number of snapshots. Moreover, since planar arrays are used, the sources can be localized in the 2-D azimuth and elevation domain, in contrast to the previous tensor based methods.

As a third contribution of this paper, we also propose to increase the spatial resolution by incorporating the proposed expanded spatial smoothing which increases the dimensions of the data tensor allowing a better identifiability while adding robustness to the proposed tensor-based methods. By exploiting the higher-order structure of the resulting expanded tensor model, a multilinear noise reduction preprocessing step via higher-order singular value decomposition is incorporated. We show that the increase on the tensor order provides a more efficient denoising, improving the performance of the proposed PARATUCK2 and Tucker4 estimators. It also yields a better performance compared to existing spatial smoothing techniques. An identifiability study is also carried out for the proposed tensor-based methods. Finally, before extracting the directions of arrival of the sources, a solution based on the multi-stage Khatri–Rao factorization [16, 23] is incorporated as a refinement stage of our proposed estimators.

The rest of this paper is organized as follows. Section 2 presents our signal model. In Section 3, we recall the baseline approach for spatial signature estimation from which the proposed methods are built. In Section 4, the proposed tensor-based estimators are formulated and the receiver algorithms are discussed. After briefly recalling conventional spatial smoothing techniques in Section 5 for completeness, we formulate the expanded spatial smoothing scheme in Section 6. A denoising procedure via a multidimensional low-rank approximation in conjunction with the expanded spatial smoothing scheme is also presented in this section. Identifiability results and computational complexity of the proposed methods are discussed in Section 7. Section 8 presents our simulation results and discussions. The paper is concluded in Section 9.

Notations: The following notation is used throughout the paper. Scalars are denoted by lower-case italic letters x, vectors are written as lower-case italic boldface letters $\boldsymbol{x}$, matrices as upper-case italic boldface letters $\boldsymbol{X}$, and tensors as calligraphic boldface letters $\boldsymbol{\mathcal{X}}$. The superscripts $^{\mathrm{T}}$, $^{\mathrm{H}}$, † and * represent transpose, Hermitian transpose, pseudo-inverse and complex conjugate, respectively. The operator $\mathrm{diag}(\boldsymbol{x})$ converts $\boldsymbol{x}$ into a diagonal matrix. The j, r-th scalar element of $\boldsymbol{X} \in \mathbb{C}^{J\times R}$ is denoted by $\boldsymbol{X}_{(j,r)}$, while its r-th column is denoted by $\boldsymbol{X}(:, r) \in \mathbb{C}^{J\times 1}$. The operator $\mathrm{vec}(\boldsymbol{X})$ converts $\boldsymbol{X}$ to a vector $\boldsymbol{x}$ by stacking its columns on top of each other, while $\mathrm{vecd}(\boldsymbol{Y})$ converts the diagonal elements of $\boldsymbol{Y}$ into a vector. The operator $\mathrm{unvec}_{J\times R}(\boldsymbol{x})$ denotes the inverse vectorization operation that converts $\boldsymbol{x} \in \mathbb{C}^{JR\times 1}$ back to a $J \times R$ matrix. $D_j(\boldsymbol{X})$ is a diagonal matrix constructed from the j-th row of $\boldsymbol{X}$, and $\|\cdot\|_F$ represents the Frobenius norm of a matrix or a tensor. The Kronecker, Khatri–Rao and Hadamard (element-wise matrix product) products are denoted by $\otimes$, $\diamond$ and $\circledast$, respectively. The Khatri–Rao product between two matrices $\boldsymbol{X} \in \mathbb{C}^{J\times R}$ and $\boldsymbol{Y} \in \mathbb{C}^{K\times R}$ corresponds to a column-wise Kronecker product, i.e.:

$$\boldsymbol{X} \diamond \boldsymbol{Y} = [\boldsymbol{X}(:, 1) \otimes \boldsymbol{Y}(:, 1), \ldots, \boldsymbol{X}(:, R) \otimes \boldsymbol{Y}(:, R)]. \tag{1}$$

In this paper, the following properties of the Khatri–Rao and Kronecker products are used

$$\mathrm{vec}(\boldsymbol{A}\boldsymbol{B}\boldsymbol{C}^{\mathrm{T}}) = (\boldsymbol{C} \diamond \boldsymbol{A})\,\mathrm{vecd}(\boldsymbol{B}), \tag{2}$$

$$\mathrm{vec}(\boldsymbol{A}\boldsymbol{B}\boldsymbol{C}^{\mathrm{T}}) = (\boldsymbol{C} \otimes \boldsymbol{A})\,\mathrm{vec}(\boldsymbol{B}), \tag{3}$$

$$(\boldsymbol{A} \diamond \boldsymbol{B})^{\mathrm{H}} (\boldsymbol{A} \diamond \boldsymbol{B}) = \boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} \circledast \boldsymbol{B}^{\mathrm{H}}\boldsymbol{B}. \tag{4}$$