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Joint state and multi-innovation parameter estimation for time-delay linear systems and its convergence based on the Kalman filtering ☆

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ABSTRACT

This paper studies the joint state and parameter estimation problem for a linear state space system with time-delay. A multi-innovation gradient algorithm is developed based on the Kalman filtering principle. To improve the convergence rate, a filtering based multi-innovation gradient algorithm is proposed by using the filtering technique. The analysis indicates that the parameter estimates given by the proposed algorithms converge to their true values under the persistent excitation conditions. A simulation example is given to confirm that the proposed algorithms are effective.

1. Introduction

Time-delays are often encountered in communication [1,2], signal processing [3,4], process control [5,6], and fault diagnosis and detection [7–9]. It is hard to avoid time-delays in industrial processes and control systems due to material transport and signal interruption [10]. The existence of time-delays makes the control system difficult to respond for changes of the inputs in time. In addition, the time-delays can trigger instability and unsatisfactory performance of the controlled processes [11]. The identification of the time-delay systems has received much research interest [12, 13]. Based on the linear regression equation, Wang et al. presented a robust instrumental variable least squares algorithm for time-delay systems from step responses [14]. Na et al. transformed the single-input single-output systems with time-delay into a parameterized form using the Taylor series expansion and developed an adaptive identification scheme [15]. The above-mentioned work was discussed for the time-delay systems with input–output representations.

Compared with the input–output representations [16–18], the state space models can reflect the motion of the inner states and involve the state estimates. A classical identification method for state space models is the subspace state space identification [19,20]. Favoreel et al. applied the subspace identification to the multi-input multi-output bilinear systems with state space representation, assuming that the inputs of the system were white and mutually independent [21]. Verdult and Verhaegen discussed the subspace identification problem of the multivariable linear parameter-varying state space systems, where the most dominant rows of the data matrices were selected for identifying the systems [22]. The subspace identification can directly provide the state space model from the input–output data, but the computational complexity increases as the dimensions of the singular value decomposition matrices and QR factorization increase.

The actual systems are usually corrupted by various stochastic noise (white noise or colored noise) and the external disturbances have important influences to signal processing [23,24] and system modeling [25,26]. In practice, it may be more reasonable to consider the disturbance by colored noise because the statistical characteristics of noises is unknown. Although the instrumental variable methods and bias correction methods are effective for identifying the systems with colored noise, these methods ignore the estimation of the parameters of the noise model [27]. The filtering technique is to eliminate the noise in the noisy measurement information and has been active in signal processing [28–30], image processing [31] and system identification [32]. This paper uses the filtering technique to study the Kalman filtering based state and parameter estimation problem. The proposed algorithms in the paper can find many potential applications in the recovery and reconstruction of measurement signals in the noisy environment. Recently, Wang and Tang developed a gradient based iterative algorithm for nonlinear systems using the data filtering [33].

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It is well-known that the Kalman filtering is based on the state space systems and is important in signal processing. This paper considers the joint state and parameter estimation problem for time-delay state space systems with colored noise by using the multi-innovation identification theory [34,35] and the filtering technique. The difficulties are that the time-delay state space system to be identified involves not only the parameters of the system model, but also those of the noise model, as well as the unknown time-delay and states. By using the multi-innovation identification theory, the system data and innovations are utilized repeatedly, and the convergence rate can be improved. The main contributions of this paper are as follows:

- By using the Kalman filtering technique, this paper presents a multi-innovation gradient (MIG) algorithm and a filtering based multi-innovation gradient (F-MIG) algorithm for estimating the states and parameters of the time-delay systems.
- By using the stochastic martingale theory, this paper analyzes the performance of the MIG algorithm and the F-MIG algorithm for time-delay state space systems.
- The proposed methods can combine the iterative technique [36–38] for studying the Kalman filtering based iterative parameter estimation approaches.

The previous work in [39] considered the state and parameter estimation for a nonlinear state space system without time-delay, by means of the key term separation technique, and did not involve convergence analysis. However, it is well-known that the time-delay system's state filtering and parameter estimation are more difficult than the no time-delay system. This paper considers the state filtering and parameter estimation for linear state space system with time-delay. Also, this paper analyzes the convergence of the proposed algorithms and given two convergence theorems and their proofs.

The remainder of this paper is organized as follows. Section 2 describes the time-delay state space systems and gives the problem statements. Sections 3 and 4 develop an MIG algorithm and an F-MIG algorithm and discuss their convergence. Section 5 provides a simulation example to test the validity of the proposed algorithms. Finally, the concluding remarks are drawn in Section 6.

2. Problem description and system model

Consider the following state space system with time-delay,

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{b}u(k), \tag{1}$$

$$y(k)=\boldsymbol{c}\boldsymbol{x}(k-\tau)+w(k), \tag{2}$$

where $\boldsymbol{x}(k):=[x_1(k),x_2(k),\cdots,x_n(k)]^{\mathrm{T}}\in\mathbb{R}^n$ represents the state vector, $u(k)\in\mathbb{R}$ and $y(k)\in\mathbb{R}$ represent the system input and output, respectively, τ is the time-delay, $w(k)\in\mathbb{R}$ is stochastic noise, $\boldsymbol{A}\in\mathbb{R}^{n\times n}$, $\boldsymbol{b}\in\mathbb{R}^n$ and $\boldsymbol{c}\in\mathbb{R}^{1\times n}$ are the system parameter matrix and vectors:

$$\boldsymbol{A}:=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\ddots&\vdots\\\vdots&\vdots&\ddots&\ddots&0\\0&0&\cdots&0&1\\a_1&a_2&a_3&\cdots&a_n\end{bmatrix}\in\mathbb{R}^{n\times n},\quad \boldsymbol{b}:=\begin{bmatrix}b_1\\b_2\\\vdots\\b_{n-1}\\b_n\end{bmatrix}\in\mathbb{R}^n,$$

$$\boldsymbol{c}:=[1,\ 0,\cdots,\ 0]\in\mathbb{R}^{1\times n}.$$

Without loss of generality, assume that $u(k)=0$, $y(k)=0$ for $k\leqslant 0$. Referring to the method in [39] and from (1), we have

$$x_i(k+1)=x_{i+1}(k)+b_iu(k),\ i=1,2,\cdots,n-1, \tag{3}$$

$$x_n(k+1)=a_1x_1(k)+a_2x_2(k)+\cdots+a_nx_n(k)+b_nu(k). \tag{4}$$

Multiplying (3) by z^{-i} and (4) by z^{-n} and using the property of the unit back shift operator z^{-1} give

$$x_i(k-i+1)=x_{i+1}(k-i)+b_iu(k-i),\quad i=1,2,\cdots,n-1, \tag{5}$$

$$x_n(k-n+1)=a_1x_1(k-n)+a_2x_2(k-n)+\cdots+a_nx_n(k-n)+b_nu(k-n). \tag{6}$$

Adding all the expressions of (5) and (6), we have

$$x_1(k)=\sum_{i=1}^{n}a_ix_i(k-n)+\sum_{i=1}^{n}b_iu(k-i). \tag{7}$$

The disturbance $w(k)$ is colored noise. Here, $w(k)$ is assumed to be an autoregressive process,

$$w(k):=\frac{1}{F(z)}v(k)\in\mathbb{R}, \tag{8}$$

where $v(k)$ is white noise with zero mean and variance σ^2, and the polynomial $F(z)$ is a function in the shift operators z^{-1} with

$$F(z):=1+f_1z^{-1}+f_2z^{-2}+\cdots+f_mz^{-m}.$$

Provided that the orders n and m are known. For the system in (1) and (2), if the state vector $\boldsymbol{x}(k)$ is known, the system matrix/vector $(\boldsymbol{A},\boldsymbol{b})$ are easy to be identified by using the similar method in [40]. This paper considers the case that the state $\boldsymbol{x}(k)$ is completely unavailable. The objective is to propose new methods for jointly estimating the unknown states and parameters from the measurement data $\{u(k),y(k)\colon k=1,2,\cdots\}$, and to study the performance of the proposed methods.

Define the parameter vectors $\boldsymbol{\vartheta}$, $\boldsymbol{\theta}$ and $\boldsymbol{f}$ as

$$\boldsymbol{\vartheta}:=[\boldsymbol{\theta}^{\mathrm{T}},\boldsymbol{f}^{\mathrm{T}}]^{\mathrm{T}}\in\mathbb{R}^{2n+m},$$

$$\boldsymbol{\theta}:=[a_1,a_2,\cdots,a_n,b_1,b_2,\cdots,b_n]^{\mathrm{T}}\in\mathbb{R}^{2n},$$

$$\boldsymbol{f}:=[f_1,f_2,\cdots,f_m]^{\mathrm{T}}\in\mathbb{R}^m,$$

and the information vectors $\boldsymbol{\varphi}(k)$, $\boldsymbol{\varphi}_s(k)$ and $\boldsymbol{\varphi}_n(k)$ as

$$\boldsymbol{\varphi}(k):=[\boldsymbol{\varphi}_s^{\mathrm{T}}(k-\tau),\boldsymbol{\varphi}_n^{\mathrm{T}}(k)]^{\mathrm{T}}\in\mathbb{R}^{2n+m},$$

$$\begin{aligned}\boldsymbol{\varphi}_s(k)&:=[x_1(k-n),x_2(k-n),\cdots,x_n(k-n),\\&\qquad u(k-1),u(k-2),\cdots,u(k-n)]^{\mathrm{T}}\\&=[\boldsymbol{x}^{\mathrm{T}}(k-n),u(k-1),u(k-2),\cdots,u(k-n)]^{\mathrm{T}}\in\mathbb{R}^{2n},\end{aligned}$$

$$\boldsymbol{\varphi}_n(k):=[-w(k-1),-w(k-2),\cdots,-w(k-m)]^{\mathrm{T}}\in\mathbb{R}^m.$$

From (7), (8) and (2), we have

$$x_1(k)=\boldsymbol{\varphi}_s^{\mathrm{T}}(k)\boldsymbol{\theta}, \tag{9}$$

$$\begin{aligned}w(k)&=[1-F(z)]w(k)+v(k)\\&=\boldsymbol{\varphi}_n^{\mathrm{T}}(k)\boldsymbol{f}+v(k),\end{aligned} \tag{10}$$

$$y(k)=x_1(k-\tau)+w(k) \tag{11}$$

$$\begin{aligned}&=\boldsymbol{\varphi}_s^{\mathrm{T}}(k-\tau)\boldsymbol{\theta}+\boldsymbol{\varphi}_n^{\mathrm{T}}(k)\boldsymbol{f}+v(k)\\&=\boldsymbol{\varphi}^{\mathrm{T}}(k)\boldsymbol{\vartheta}+v(k).\end{aligned} \tag{12}$$

The information vector $\boldsymbol{\varphi}(k)$ consists of the state vector $\boldsymbol{x}(k-\tau-n)$, the input $u(k-i)$ and the correlated noise $w(k-i)$, and the parameter vector $\boldsymbol{\vartheta}$ consists of the parameters a_i and b_i of the state space model in (1)–(2) and the parameters f_i of the noise model in (8).