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Locality-constrained nonnegative robust shape interaction subspace clustering and its applications ☆

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ABSTRACT

In this paper, we present a locality-constrained nonnegative robust shape interaction (LNRSI) subspace clustering method. LNRSI integrates the local manifold structure of data into the robust shape interaction (RSI) in a unified formulation, which guarantees the locality and the low-rank property of the optimal affinity graph. Compared with traditional low-rank representation (LRR) learning method, LNRSI can not only pursuit the global structure of data space by low-rank regularization, but also keep the locality manifold, which leads to a sparse and low-rank affinity graph. Due to the clear block-diagonal effect of the affinity graph, LNRSI is robust to noise and occlusions, and achieves a higher rate of correct clustering. The theoretical analysis of the clustering effect is also discussed. An efficient solution based on linearized alternating direction method with adaptive penalty (LADMAP) is built for our method. Finally, we evaluate the performance of LNRSI on both synthetic data and real computer vision tasks, i.e., motion segmentation and handwritten digit clustering. The experimental results show that our LNRSI outperforms several state-of-the-art algorithms.

1. Introduction

In the past decades, subspace learning and clustering problems have been an interesting research topic in computer vision and machine learning. Generally, it is assumed that the data points are drawn from multiple low dimensional subspaces, and then the basic task of subspace clustering is to cluster the data points according to the underlying subspace.

A number of subspace clustering methods have been developed. According to the mechanism of clustering, these methods can be divided into 4 groups: algebraic methods ([1,2]), iterative methods ([3,4]), statistical methods ([5–7]), and spectral clustering methods ([8,9]). Due to the fruitful theory and simple implement, spectral clustering has become one of the most popular subspace clustering algorithms ([10]).

The key step of spectral clustering is the construction of the affinity graph. In general, the ways to construct affinity graph can be classified into the local distance based scheme and the global linear representation based scheme. Traditional local methods adopt the Euclidean distance between pairwise points to build the similarity graph, such as Laplacian Eigenmaps [11], k-NN [12], LLE [13]. If the data X_j is not located within the neighbors of X_i, then the elements of affinity matrix are set as $Z_{ij} = 0$, otherwise, Z_{ij} will be determined by the distance between data points X_i and X_j. The local method can capture the local structure of the datasets, and it's sparse and discriminative. But it ignores the global characteristic of the entire data set, so it's sensitive to noise and outliers. Generally, local methods work well if the neighbors for each sample point are correctly selected, i.e., the selected neighbors can reflect the local geometric structure of the manifold subspace.

Compared with local distance based method, the global linear representation based schemes assume that each data point can be linearly represented in an over-complete dictionary (i.e., $X = DZ$, where D is the overcomplete dictionary). By applying different kinds of regularization on the representation space, various affinity graph can be constructed, such as sparse subspace clustering (SSC), linear squared regression (LSR), low rank representation (LRR), etc.

Sparse subspace clustering [8,14] assume that a data point in the union of multiple subspaces admits a sparse representation with respect to the dictionary formed by all other data points, i.e., $\min \|Z\|_0,\ s.t.\ X = XZ$. It is also shown that, under the assumption

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that the subspaces are independent, if $Z_{ij} \neq 0$, then data X_i and X_j will belong to the same subspace. Finally, the affinity graph can be constructed by the sparse representation matrix Z. SSC have achieved many state-of-art results in several applications, such as feature extraction [15,16], motion segmentation [17,18]. However, it suffers from the following problems. On one hand, SSC is actually a point-wise analysis. It can be divided into n independent subproblems $\min \sum_i \|Z_i\|_0,\ s.t.\ X_i = XZ_i$. So the correlation between data and the global characteristic of the subspace may be ignored. On the other hand, if there are several data points having strong linear correlation with X_i, the SSC will randomly choose one. Therefore, while SSC can pursue the sparsity of the affinity graph, it may reduce the accuracy of clustering.

Compared with sparse based methods, low-rank based methods aim at finding the lowest rank representation of all data jointly, i.e., $\min rank(Z),\ s.t.\ X = XZ$. Therefore, it's more suitable to pursue the global and intrinsic information of data space. If the data points are sampled from a single subspace, the underlying subspace structure can be exactly recovered by the Robust Principle Component Analysis (RPCA) [19] under some mild conditions. The work by Liu [9] extends the recovery of corrupted data from single subspace to multiple subspaces, and finds that the multiple subspace structure can be revealed by the low rank representation (LRR) coefficients of a given dictionary. As the rank regularization is not convex and difficult to optimize, the nuclear norm is usually used as the convex surrogate of rank. Low-rank based methods have been successfully applied to applications including salient object detection [20], segmentation and grouping [21], background subtraction [22], tracking [23], and 3D visual recovery [24]. However, LRR is unable to utilize the underlying local linear structure of data, which will result in that the constructed affinity matrix is usually dense and not block diagonal even under certain strong assumptions, e.g. independent subspaces.

According to the above analysis, the local distance based methods, e.g. k-NN, utilize the locality structure of data but lack the global property, while for the LRR method, the case is contrary. Ideally, if we assume that the subspaces are independent, the affinity matrix should be close to block diagonal sparse matrix, with nonzero entries of each block corresponding to the data point pairs from the same subspace. In another word, the affinity graph should be both local and low-rank. Therefore, in this paper, we present an locality-constrained robust shape interaction (LNRSI) learning algorithm. LNRSI integrates the local manifold structure of data into the low rank representation in a unified formulation, which lead to a sparse and low-rank affinity matrix. Compared with traditional low rank representation learning method, LNRSI can not only pursuit the global structure of data space by rank regularization, but also keep the locality manifold which is discriminative.

Compared with existing works, the main contributions of this paper include:

1. The proposed algorithm which integrates the local structure with the global robust shape interaction, guarantees the locality and the low-rank property of the optimal affinity graph;
2. In this paper, we give the theoretic analysis of the clustering effect of the proposed algorithm, and we prove that the optimal affinity matrix obtained by our algorithm will be block-diagonal under some mild conditions.
3. An efficient solution based on linearized alternating direction method with adaptive penalty (LADMAP) is built for our method.

The remainder of this paper is organized as follows: Section 2 reviews the related work. In section 3, the locality-constrained nonnegative robust shape interaction model is proposed, and the theoretical analysis is given to guarantee the good grouping effect. We also propose a practical solution by applying linearized alternating direction method with adaptive penalty in section 3. The relationship with existing work is described in section 4 and the experimental results are shown in section 5. Finally, the concluding and remarks are given in section 6.

2. Related works

Given a set of the data points $X = [X_1,\ X_2,\ \cdots,\ X_n] \in R^{m\times n}$, where each sample $X_i \in R^m$ is drawn from a subspace $\mathcal{S}_k$, $k = 1, \cdots, p$, where $\mathcal{S}_k$ means the k-th subspace, and p is the number of the different subspaces. The task of clustering is to cluster the data points X_i to their corresponding subspace correctly.

2.1. Robust Principle Component Analysis (RPCA)

The RPCA aims to recover the low-rank matrix X_0 from the given observation matrix X corrupted by errors E_0 ($X = X_0 + E_0$). Motivated by the advances in low-rank matrix analysis, RPCA can be solved by solving the following regularized rank minimization problem:

$$\min_{D,E} rank(D) + \lambda\|E\|_0,\ s.t.\ X = D + E$$

where λ is arbitrary balanced parameter. This problem is NP-hard, and the research work [4] shows that if the rank of D is not too large and E is sparse, the optimization problem is equivalent to:

$$\min_{D,E} \|D\|_* + \lambda\|E\|_1,\ s.t.\ X = D + E$$

where $\|\cdot\|_*$ means the nuclear norm, which is the best convex envelope of the rank. The work [19] shows that under fairly general conditions, D can be exactly recovered from X as long as E is sufficiently sparse (relative to the rank of D).

RPCA has been successfully applied to many machine learning and computer vision problems, such as video surveillance [25], face modeling [26], etc. However, the RPCA implicitly assumes that the underlying data structure is a single low-rank subspace. When the data is drawn from a union of multiple subspaces, $\mathcal{S}_k, k = 1, \cdots, p$, it actually treats the data as being sampled from a single subspace defined by $S = \bigcup_{k=1}^{p} \mathcal{S}_k$.

2.2. Low rank representation (LRR)

LRR aims at finding the lowest-rank representation of a collection of vectors jointly. It can be seen as a generalization of RPCA. LRR could handle well the data drawn from a union of multiple subspaces. Compared with RPCA, LRR better captures the multiple subspace structure, and gives a more effective tool for robust subspace clustering.

For the noiseless case, LRR takes the data itself as a dictionary and seeks the representation matrix with the lowest rank.

$$\min_{Z} rank(Z),\ s.t.\ X = XZ$$

Due to the fact that the rank function is not convex and difficult to be optimized, the above optimization problem can be relaxed to the following convex optimization:

$$\min_{Z} \|Z\|_*,\ s.t.\ X = XZ$$

where $\|\cdot\|_*$ means the nuclear norm. The nuclear norm of a matrix equal to the sum of the singular values of this matrix.

LRR can also handle noisy data by adding a $\ell_{2,1}$-norm term to the objective function in order to make the noise column sparse.

$$\min_{Z} \|Z\|_* + \lambda\|E\|_{2,1},\ s.t.\ X = XZ + E$$