



Gridless compressive sensing method for line spectral estimation from 1-bit measurements



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ABSTRACT

This paper considers the problem of recovering frequency sparse signals which consist of a few complex sinusoids and estimating the frequency components from 1-bit quantized measurements. Unlike previous grid-based 1-bit compressive sensing approaches, we present a gridless convex method to recover frequency sparse signals from 1-bit measurements via binary atomic norm minimization (BANM). And the frequencies can take any continuous values in the frequency domain, which overcomes grid mismatches caused by the off-grid problem. We further propose a dual polynomial method to achieve continuous frequency estimation. Moreover, we present an efficient algorithm to solve BANM for large scaled problem. Numerical experiments are performed to demonstrate the effectiveness of our method compared with the grid-based compressive sensing algorithm.

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1. Introduction

Spectral analysis of signals which are superposition of complex exponentials is a fundamental problem in statistical signal processing. The line spectral estimation problem plays an important role in wide applications such as radar imaging, communications, spectroscopy, astronomy and so on. In this paper, we consider a signal to be observed is constructed as follows:

$$x'_j = \sum_{k=1}^K s_k e^{i2\pi j f_k}, \quad j = 0, \dots, N-1 \quad (1)$$

where $i = \sqrt{-1}$, $f_k \in [0, 1)$ denotes the normalized frequency to be estimated and $s_k \in \mathbb{C}$ represents the unknown complex amplitude of the k th sinusoidal component. The model order $K < N$ is usually unknown as well in practice. Many methods have been proposed to deal with the frequency estimation problem. Polynomial interpolation using Prony's method can exactly extract the frequency components from $2K$ samples. However, the performance is often sensitive to the polynomial root finding. The nonlinear least squares (NLS) method is based on the nonlinear regression model [1]. It determines the unknown parameters by minimizing a non-convex function, which is hard to provide guarantee of global optimization. Multiple signal classification (MUSIC) is another classic

method for frequency estimation, which is based on the eigenvalue decomposition of covariance of the signal [2]. The methods mentioned above also require the information of the model order K , which is not easy to obtain in practice.

In the past decade, compressive sensing (CS) theory has achieved significant success in signal processing area and information theory [3]. Based on sparse signal representation, several methods have been developed for frequency estimation. In this kind of methods, the signal has to be sparsely represented under a finite dictionary. So the continuous frequency component has to be discretized into a finite set of grids, and the true frequencies are assumed to be on the grid points. However, if the frequencies do not fall onto the grid point, the basis mismatch problem will lead to the degradation of the performance [4,5]. Although using dense grid will alleviate the basis mismatch, it will lead to higher coherent matrix which violates the restricted isometry property (RIP) [6]. There are also many grid-based methods to deal with this off-grid problem. Many of them jointly estimate the new grid during the algorithms [7–9]. Despite of the grid-based algorithms, several gridless sparse methods have been proposed for line spectral estimation recently. Motivated by the concept of atomic norm [10, 11] provides exactly recovered condition for noiseless case. In the presence of missing data, an atomic norm minimization method is introduced in [12]. In [13–15], the noisy case is studied. In particular, in the noisy case with incomplete data, [15] proposes a gridless version of SPICE [16] for line spectral estimation.

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The conventional CS framework assumes that the observation of the signal is linear, i.e.

$$\mathbf{y} = \mathbf{A}\mathbf{x}' \quad (2)$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$ is the measurement matrix, and $\mathbf{y} \in \mathbb{C}^M$ is the measurement vector. Thus each of the measurements has infinite bit precision. In practice, the measurements have to be quantized to finite bit depth. In the quantized compressive sensing (QCS), we have

$$\mathbf{y} = Q(\text{Re}(\mathbf{A}\mathbf{x}')) + i \cdot Q(\text{Im}(\mathbf{A}\mathbf{x}')) \quad (3)$$

where $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ return the real part and imaginary part of a vector respectively, and $Q(\cdot)$ performs quantization operator element-wisely on the vector. Thus, each continuous value is mapped to a discrete value in a finite set. The effect of quantization is studied by [17–19].

Furthermore, recent studies have shown that stable signal recovery can be guaranteed even if the measurements are quantized to one bit. In this case, the analog-to-digital converter (ADC) becomes a comparator, which is fast, inexpensive, with the characteristics of low power consumption and robust to amplification distortion. These appealing benefits have attracted many studies on it. In this setting, the observation of the signal becomes

$$\mathbf{y} = \text{sign}(\text{Re}(\mathbf{A}\mathbf{x}')) + i \cdot \text{sign}(\text{Im}(\mathbf{A}\mathbf{x}')) \quad (4)$$

where $\text{sign}(z) = z/|z|$ performs element-wisely on the vector for $z \neq 0$ and define $\text{sign}(0) = 1$. Since the scaling information of the measurements is totally lost, the goal is to recover the signal on the unit hyper-sphere. The 1-bit CS framework was first studied in [20]. Following that, many approaches have been developed, including greedy method [21], hard thresholding method [22], convex relaxation method [23,24], trust region method [25], Bayesian inference [26,27] and other methods [28,29]. The methods mentioned above all assume that the signal can be sparsely represented by a discrete dictionary, which suffers from the basis mismatch problem in practice.

In this paper, we consider the problem of recovering a frequency sparse signal and extracting its frequency components from 1-bit quantized measurements. Motivated by the studies on atomic norm, we develop a new gridless 1-bit compressive sensing method for line spectral estimation. We propose a convex approach for signal recovery, denoted as binary atomic norm minimization (BANM). We also develop a dual polynomial method to extract the frequency components of the signal, which is efficient and does not require the model order as prior information. Furthermore, we provide an effective method for solving BANM via the Alternating Direction Method of Multipliers (ADMM) [30].

The rest of the paper is organized as follows. Section 2 introduces the convex gridless approach for 1-bit spectral compressive sensing. Section 3 provides the dual polynomial method to achieve the estimation of the continuous-valued frequency components. Section 4 presents an effective algorithm for solving BANM. Numerical experiments are provided in Section 5 to verify the performance of BANM, and we make conclusions of this paper in Section 6.

2. Binary atomic norm minimization for continuous compressive sensing

Let us first recall the existing grid-based 1-bit CS methods for real-valued signal. For a real-valued sparse signal $\mathbf{z} \in \mathbb{R}^N$ and a measurement matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$, the observation can be described as

$$\mathbf{y} = \text{sign}(\mathbf{A}\mathbf{z}) \quad (5)$$

In order to recover $\mathbf{z}$ from the measurement vector $\mathbf{y}$, the optimization problem can be formulated as

$$\begin{aligned} \hat{\mathbf{z}} &= \arg \min \|\mathbf{z}\|_0 \\ \text{s.t. } & \text{sign}(\mathbf{A}\mathbf{z}) = \text{sign}(\mathbf{y}) \\ & \|\mathbf{z}\|_2^2 = 1 \end{aligned} \quad (6)$$

The optimization problem (6) is NP-hard and the constraint $\|\mathbf{z}\|_2^2 = 1$ is nonconvex, which makes it intractable to solve. In [23], Plan provided a convex approach to achieve stable recovery:

$$\begin{aligned} \hat{\mathbf{z}} &= \arg \min \|\mathbf{z}\|_1 \\ \text{s.t. } & \mathbf{Y}\mathbf{A}\mathbf{z} \geq 0 \\ & \|\mathbf{A}\mathbf{z}\|_1 = 1 \end{aligned} \quad (7)$$

where $\mathbf{Y} = \text{diag}(\mathbf{y})$ denotes the diagonal matrix with $\mathbf{y}$ being its diagonal elements. Although the algorithm is designed for solving the real-valued problem, it can be easily extended to complex-valued case.

Suppose a spectral sparse signal $\mathbf{x}'$ is defined as (1). Consider the problem of recovering $\mathbf{x}'$ from the 1-bit quantized measurement $\mathbf{y}$. The quantization is described in (4). In order to utilize the grid-based method, we first discretize the continuous frequency domain $[0, 1)$ into finite grids. Let $\mathbf{D} \in \mathbb{C}^{N \times P}$ be the discrete sparse dictionary with element of m row n column being

$$D_{m,n} = e^{i2\pi(m-1)(n-1)/P} \quad (8)$$

Then we assume $\mathbf{x}'$ can be sparsely represented as

$$\mathbf{x}' = \mathbf{D}\boldsymbol{\theta} \quad (9)$$

where $\boldsymbol{\theta}$ represents the discretized frequency components of $\mathbf{x}'$. In a similar manner, we can extend the grid-based convex method to complex-valued case:

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg \min \|\boldsymbol{\theta}\|_1 \\ \text{s.t. } & \text{Re}(\mathbf{Y}) \cdot \text{Re}(\mathbf{A}\mathbf{D}\boldsymbol{\theta}) \geq 0 \\ & \text{Im}(\mathbf{Y}) \cdot \text{Im}(\mathbf{A}\mathbf{D}\boldsymbol{\theta}) \geq 0 \\ & \|\text{Re}(\mathbf{A}\mathbf{D}\boldsymbol{\theta})\|_1 + \|\text{Im}(\mathbf{A}\mathbf{D}\boldsymbol{\theta})\|_1 = 1 \end{aligned} \quad (10)$$

The objective function encourages a sparse solution of $\boldsymbol{\theta}$. The first and second constraints enforce the consistency with the measurements, and the last constraint serves to prevent the trivial zero solution and also makes (10) a convex problem. We denote (10) as grid-based l_1 method. Once the optimal value $\hat{\boldsymbol{\theta}}$ is obtained, we get the estimation of $\hat{\mathbf{x}}$ by $\hat{\mathbf{x}} = \mathbf{D}\hat{\boldsymbol{\theta}}$.

As discussed before, this grid-based method also suffers from the basis mismatch problem if the true frequencies do not coincide with the discrete sparse dictionary $\mathbf{D}$. Recently, motivated by the concept of atomic norm [10], some gridless methods have been studied. Let $\mathcal{A}$ be the set of atoms which construct the signal. The atomic norm $\|\cdot\|_{\mathcal{A}}$ is defined by identifying its unit ball with the convex hull of $\mathcal{A}$

$$\begin{aligned} \|\mathbf{x}\|_{\mathcal{A}} &= \inf\{t > 0 : \mathbf{x} \in t \text{conv}(\mathcal{A})\} \\ &= \inf\left\{\sum_k c_k : \mathbf{x} = \sum_k c_k \mathbf{a}_k, c_k \geq 0, \mathbf{a}_k \in \mathcal{A}\right\} \end{aligned} \quad (11)$$

and the dual norm of the atomic norm is given by

$$\|\mathbf{x}\|_{\mathcal{A}}^* = \sup\{\langle \mathbf{x}, \mathbf{a} \rangle_{\mathcal{R}} : \mathbf{a} \in \mathcal{A}\} \quad (12)$$

The real inner product $\langle \mathbf{x}, \mathbf{a} \rangle_{\mathcal{R}}$ is defined by $\langle \mathbf{x}, \mathbf{a} \rangle_{\mathcal{R}} = \text{Re}(\langle \mathbf{x}, \mathbf{a} \rangle) = \text{Re}(\mathbf{a}^H \mathbf{x})$, where the superscript H denotes the conjugate transpose. In our application, the set of atoms $\mathcal{A}$ is given by

$$\mathcal{A} = \{\mathbf{a}(f, \phi) : f \in [0, 1), \phi \in [0, 2\pi)\} \quad (13)$$

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