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Hierarchical distributed compressive sensing for simultaneous sparse approximation and common component extraction ☆

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ABSTRACT

Simultaneous sparse approximation is a generalization of the standard sparse approximation, for simultaneously representing a set of signals using a common sparsity model. Distributed compressive sensing (DCS) framework has utilized simultaneous sparse approximation for generalizing compressive sensing to multiple signals. DCS finds the sparse representation of multiple correlated signals from compressive measurements using the common + innovation signal model. However, DCS is limited for joint recovery of a large number of signals since it requires large memory and computational time. In this paper, we propose a new hierarchical algorithm to implement the joint sparse recovery part of DCS more efficiently. The proposed approach is based on partitioning the input set and hierarchically solving for the sparse common component across these partitions. The numerical evaluation of the proposed method shows the decrease in computational time over DCS with an increase in reconstruction error. The proposed algorithm is evaluated for two different applications. In the first application, the proposed method is applied to video background extraction problem, where the background corresponds to the common sparse activity across frames. In the second application, a common network structure is extracted from dynamic functional brain connectivity networks.

1. Introduction

Sparse signal approximation refers to the representation of a signal as a linear combination of a small subset of elements of a dictionary [1]. Sparse signal approximation usually considers one signal at a time, not taking into account the correlation within a group of signals. Simultaneous sparse approximation, *a.k.a.* joint sparse recovery, on the other hand, finds sparse representations of multiple signals collected through sensors monitoring the same environment simultaneously using a common dictionary [2–4]. Joint sparse recovery has been used in many applications such as sensor networks [5], neuroelectromagnetic imaging [6,7], source localization [8], and image restoration [9].

Different approaches to finding the common sparse representation among a set of signals have been proposed. Tropp et al. [2] proposed a greedy algorithm, *i.e.* simultaneous orthogonal matching pursuit (S-OMP), which extends orthogonal matching pursuit to joint sparse recovery. In [3], a convex relaxation approach was used to find the joint sparse approximation of multiple signals. Blanchard et al. [10] extended well-known sparse approximation methods, iterative hard thresholding, normalized iterative hard thresholding, hard thresholding pursuit, normalized hard thresholding pursuit, and Compressive Sampling Matching Pursuit, to the joint sparse recovery problem.

A closely related line of work was proposed in the compressive sensing community. Distributed compressive sensing (DCS) is an extension of compressive sensing to multiple observations problem [11,12]. It simultaneously finds the sparse representation of a set of compressively sensed signals by assuming a common + innovation component model for signals. The common + innovation component model makes DCS a suitable tool to extract common component of highly correlated signals in many applications such as video processing [13–15] and time-varying networks [16]. However, a major problem with using DCS to extract the common component of a set of signals is that the size of the dictionary increases dramatically with the number of signals. For a set of J N-dimensional signals, the size of the required dictionary in DCS method to find the common component is $JN \times (J+1)N$, which demands huge memory and computational resources. In the joint sparse recovery literature, several methods for addressing this high computational complexity have been introduced. Lee et al. [17] proposed orthogonal subspace matching pursuit (OSMP) for a new joint sparse recovery method, SA-MUSIC. OSMP is used in the partial support recovery step of SA-MUSIC to provide a com-

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putationally efficient solution to joint sparse recovery with a performance guarantee. In [18], the performance of the joint sparse recovery of infinite measurement vectors (IMV), an infinite set of jointly sparse vectors, is improved. Instead of discretizing the entire infinite set of sparse vectors in IMV models, a reduced finite-dimensional problem is derived from which the common nonzero location set can be inferred exactly. In [19], $l_{1,2}$-norm penalty is used to enforce joint sparsity on the signals. The accelerated proximal gradient finds efficiently the joint sparse representation of the set of signals. Hyder and Mahata [20] proposed joint $l_{2,0}$ approximation algorithm (JLZA), an extension of the zero-norm approximation algorithm, to decrease the computational complexity while ensuring the robustness to the measurement noise. Even though these algorithms have improved the computational complexity of simultaneous sparse recovery problem, they all assume that the set of signals is sparse with respect to the same dictionary and the locations of the nonzero entries of the vector coefficient are the same among all signals while their values are different. However, DCS focuses on a more general problem with the assumption that the collection of signals have a common sparse component with respect to a basis while the innovation components are sparse with respect to another dictionary with the locations of the nonzero entries of the coefficient vector being different. Thus, these methods are not directly applicable to DCS.

In this paper, we propose a hierarchical algorithm to implement the joint sparse recovery part of DCS more efficiently. The proposed algorithm, called hierarchical distributed compressive sensing (H-DCS), partitions the set of signals into a small number of subsets, and finds the common component of each subset separately. The common components of the subsets are then used as a new set of signals to find the common component across all signals. The error bound and the computational complexity of DCS and hierarchical DCS are derived and compared, showing how hierarchical DCS outperforms DCS in terms of computational complexity. However, the error bound of hierarchical DCS is slightly larger than DCS due to the aggregated approximation errors across the iterations. Although this paper is focused on the two-stage implementation of H-DCS, it can be easily extended to more stages.

The proposed algorithm is evaluated for two different applications. In the first application, we consider the video background extraction problem which is of great importance in many automatic video content analysis applications such as surveillance video coding [21], motion detection [22], object tracking [23,24], *etc.* Video background extraction aims to separate the moving objects, *a.k.a.* foreground, from the static objects, *a.k.a.* background, in order to facilitate the tracking of moving objects. Even though the subsequent steps of the background extraction are of more interest, the accuracy of the overall system depends on the performance of background extraction. Since the background scene does not change noticeably over time, it has a sparse representation with respect to a frequency basis such as DCT. Thus, H-DCS with discrete cosine transform (DCT) basis is used to extract the background scene.

In the second application, a method based on H-DCS is proposed to track dynamic functional connectivity brain networks over time. Functional connectivity (FC) has been extensively used to understand cognitive brain processes [25,26]. One way to study FC networks in detail is through neuronal time series recorded from electroencephalogram (EEG) [25]. These time series are transformed to connectivity networks through bivariate synchronization measures between brain regions [27], where nodes correspond to distinct brain regions and edges to functional connectivity between them [28–30]. Evidence points to the fact that FC networks continuously form and destruct over multiple short-time intervals due to task demands, learning and anesthesia [31–33]. It is shown that these time-varying FC networks consist of a small number of distinct FC states corresponding to quasi-stationary time intervals [34,35]. Thus, one way to track the dynamics of time-varying FC networks is to detect the change points in time where the FC networks change. FC networks generally consist of a background activity, which is common across all time steps, and a foreground network which corresponds to the transient activations [36]. The background activity is similar to the default mode network (DMN) identified from resting state fMRI, and varies slowly across time such that it can be assumed to be sparse with respect to frequency domain bases. Thus, the background activity (background FC pattern) is separated from the foreground by applying H-DCS with DCT as the basis of the common component. Since the innovation components are unique to each FC network, the dissimilarity of the innovation components of the consecutive FC networks is employed to detect the change points in the network structure. Once change points are detected, the time intervals can be summarized to obtain their common FC pattern through another stage of H-DCS algorithm.

2. Distributed compressive sensing

Distributed compressive sensing assumes that signals acquired across multiple sensors are sparse in a collection of bases, *i.e.* the set of signals is jointly sparse. Due to the inter-signal correlation, jointly sparse signals are usually assumed to be composed of a common sparse component which is shared by all signals, and an innovation component which is unique to each signal [37]. The encoding part of distributed compressive sensing is not different from compressive sensing in that each signal is separately projected onto some random, incoherent bases. However, the decoding is based on simultaneous sparse recovery of all signals, which can be used for various purposes including the common component extraction. In this paper, we only focus on the sparse recovery part of distributed compressive sensing.

2.1. Joint sparsity model

Let's assume that the set of signals $\Lambda = \{\mathbf{x}_j \in \mathbb{R}^N; \forall j \in \{1, 2, \ldots, J\}\}$ are jointly sparse. It is assumed that there is an inter-signal correlation among the signals. The joint sparsity model (JSM) [38,12], which includes a common component $\mathbf{z}_c \in \mathbb{R}^N$ and an innovation component $\mathbf{z}_j \in \mathbb{R}^N$, can be written as:

$$\mathbf{x}_j = \mathbf{z}_c + \mathbf{z}_j, \quad j = 1, 2, \ldots, J. \tag{1}$$

The common component represents the inter-signal correlation among the signals while the innovation component is unique to each signal. The common and innovation components of the set of signals Λ are sparse with respect to two different sets of bases, $\boldsymbol{\phi}_c$ and $\boldsymbol{\phi}_j$, respectively, as:

$$\begin{aligned} \mathbf{z}_c &= \boldsymbol{\phi}_c \boldsymbol{\theta}_c, \\ \mathbf{z}_j &= \boldsymbol{\phi}_j \boldsymbol{\theta}_j, \qquad j \in \{1, 2, \ldots, J\}, \end{aligned} \tag{2}$$

where $\boldsymbol{\theta}_c$ and $\boldsymbol{\theta}_j$ are the coefficient vectors, and the bases $\boldsymbol{\phi}_c$ and $\boldsymbol{\phi}_j$ are orthogonal. Since the signal $\mathbf{x}_j$ is sparse, the coefficient vectors have a small number of nonzero entities, $\|\boldsymbol{\theta}_c\|_0 = K_c \ll N$ and $\|\boldsymbol{\theta}_j\|_0 = K_j \ll N$.

In order to recover the sparse representation of the set of signals Λ, all signals are stacked to form a single optimization problem. Eq. (3) shows the compact representation of all signals and their sparse representations in matrix format.

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