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A robust DOA estimator based on the correntropy in alpha-stable noise environments

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ABSTRACT

By introducing correntropy as the robust statistics, a novel direction of arrival estimator for α-stable noise is proposed. In this method, the signal subspace is estimated by solving the correntropy based optimization problem under the maximum correntropy criterion. An optimal step size based iterative algorithm is developed and the convergence of it is proved. Comprehensive simulation results demonstrate that the proposed method is superior to several existing algorithms in terms of the probability of resolution and the estimation accuracy, especially in the highly impulsive noise environments.

1. Introduction

Source localization by direction of arrival (DOA) has been received considerable attention in radar, sonar and wireless communication over the past years [1]. The multiple signal classification (MUSIC) algorithm [2], one of the most well-known subspace methodology, has attracted much attention and is an asymptotically unbiased DOA estimator based on the Gaussian noise assumption [3,4]. However, in some scenarios, noise exhibits an impulsive nature mainly characterized by sudden bursts or sharp spikes [5,6], and it is inappropriate to model the noise as Gaussian distributed.

It has been shown that the impulsive noise could be better modeled by α-stable distribution [7]. To overcome the α-stable noise, a series of DOA estimation algorithms based on the fractional lower-order statistics (FLOS) were proposed [8–12]. However, the algorithms in [8–10] are only robust to circularly symmetrical signals, and the characteristic exponent of noise must be estimated to ensure $1 \le p < \alpha$ [8,9,12], $0 < p < \alpha/2$ [10] or $0 < p < \alpha$ [11], where p is the order of moments and α is the characteristic exponent of noise. Another scheme is named as data-adaptive zero-memory (DA-ZM) algorithm [13–18] which exploits the zero-memory nonlinear functions to suppress the outliers. Compared with the FLOS-based algorithms, the DA-ZM algorithms can provide more accurate and robust DOA estimates and need shorter snapshots. However, some DA-ZM methods [15–18] may cross-correlate the signal subspace and noise subspace which leads to the performance degradation [13]. The Huber's minimax robust estimation theory based beamforming methods [19–22] were also developed for combating impulsive noise. Different from the methods listed above, the signal subspace was directly estimated by minimizing the Lp norm ($1 \le p < 2$) of the residual fitting error matrix in ACO-MUSIC [23]. Although it is superior to several existing outlier-resistant methods, the performance of it will degrade significantly in highly impulsive noise environments.

Correntropy has been proposed as a new robust statistics that can quantify both the time structure and the statistical distribution of two stochastic processes and is inherently robust to outliers [24]. Although the correntropy has been adopted in several problems [25–27], it is rarely used in the DOA estimation problems, because it does not induce a homogeneous moment in the sample space [24,28]. To the best of our knowledge, the existing DOA estimation methods based on the correntropy were studied in [28,29]. Unfortunately, in these methods above, the correntropy is only used as the weighting coefficient to obtain the bounded covariance matrix, which retains the conventional correlation term and is bounded if and only if the characteristic exponent satisfies $1 < \alpha \le 2$ [28]. The performance will degrade when the underlying noise is highly impulsive.

In this paper, a robust DOA estimator is proposed by adopting the correntropy as the robust statistics, which can realize more robust and accurate DOA estimates. The main contributions of this paper are listed as: (1) a novel DOA estimator based on correntropy is proposed, in which the signal subspace is estimated by solving the optimization problem under the maximum correntropy criterion (MCC); (2) an optimal step size based stochastic gradient algorithm is designed to solve the optimization problem, and

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the convergence of the iterative algorithm is proved; and (3) a better performance of the novel DOA estimator is demonstrated compared with several existing methods.

The rest of this paper is organized as follows. In Section 2, the signal model for DOA estimation is given, the definition and properties of symmetric α-stable distribution ($S\alpha S$) and correntropy are presented briefly. In Section 3, a novel correntropy based DOA estimator is proposed and an iterative algorithm is developed. In section 4, simulation results are presented to demonstrate the effectiveness of the proposed method. Finally, some concluding remarks are drawn in Section 5.

2. Problem formulation

2.1. Problem definition

Consider P independent far-field narrow-band sources with known frequency impinging on a uniform linear array (ULA) composed of M $(P < M)$ identical omni-directional sensors with inter-element spacing d. For a given snapshot, the signal observed by the m-th sensor can be modeled as

$$x_m(t) = \sum_{i=1}^{P} s_i(t) \mathrm{e}^{\mathrm{j}2\pi \sin\theta_i (m-1)d/\lambda} + n_m(t) \tag{1}$$

where θ_i is the DOA and $s_i(t)$ is the complex envelop of the i-th source, λ is the wavelength and $n_m(t)$ is the additive α-stable noise.

The vector form for (1) can be expressed as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{2}$$

where $\boldsymbol{x}(t) = [x_1(t), \cdots, x_M(t)]^{\mathrm{T}}$ is the received signal vector, $\boldsymbol{s}(t) = [s_1(t), \cdots, s_P(t)]^{\mathrm{T}}$ is the source signal vector, $\boldsymbol{n}(t) = [n_1(t), \cdots, n_M(t)]^{\mathrm{T}}$ is the noise vector, and $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_P)]$ is the array manifold matrix with steering vector written as

$$\boldsymbol{a}(\theta_i) = \left[1, \cdots, \mathrm{e}^{\mathrm{j}2\pi \sin\theta_i (m-1)d/\lambda}, \cdots, \mathrm{e}^{\mathrm{j}2\pi \sin\theta_i (M-1)d/\lambda}\right]^{\mathrm{T}} \tag{3}$$

Our goal is to estimate the DOAs of the multiple independent sources from the received array data.

2.2. $S\alpha S$ distributions

A complex random variable $X = X_1 + \mathrm{j}X_2$ is $S\alpha S$ if and only if X_1 and X_2 are jointly $S\alpha S$, and it is isotropic if and only if (X_1, X_2) has a uniform spectral measure [8]. In this case, the characteristic function of random X can be expressed as

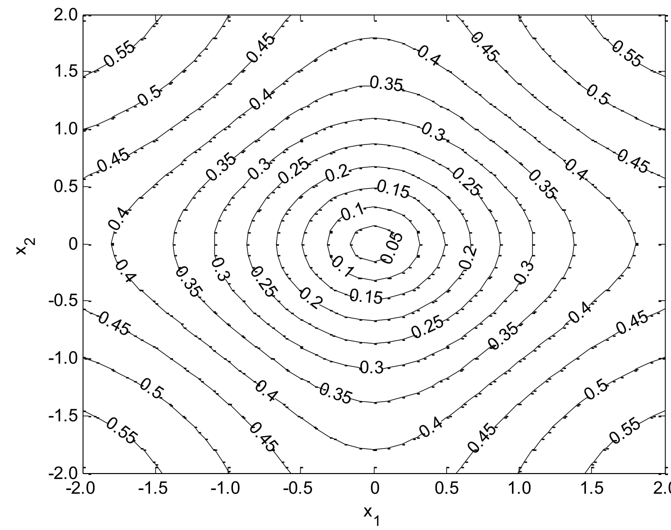
$$\varphi(\omega) = \exp\left\{\mathrm{j}\mu\omega - \gamma|\omega|^{\alpha}\right\} \tag{4}$$

where α $(0 < \alpha \le 2)$ is the characteristic exponent, γ $(\gamma > 0)$ is the dispersion parameter, μ $(-\infty < \mu < +\infty)$ is the location parameter (see [7] and the references therein for detail). When $\alpha = 2$, the $S\alpha S$ distribution reduces to a Gaussian distribution, and when $\alpha = 1$, the $S\alpha S$ distribution becomes a Cauchy distribution. Because the second-order and higher-order moments of α-stable processes with $\alpha < 2$ are infinite, the performance of conventional MUSIC method degrades significantly under α-stable noise.

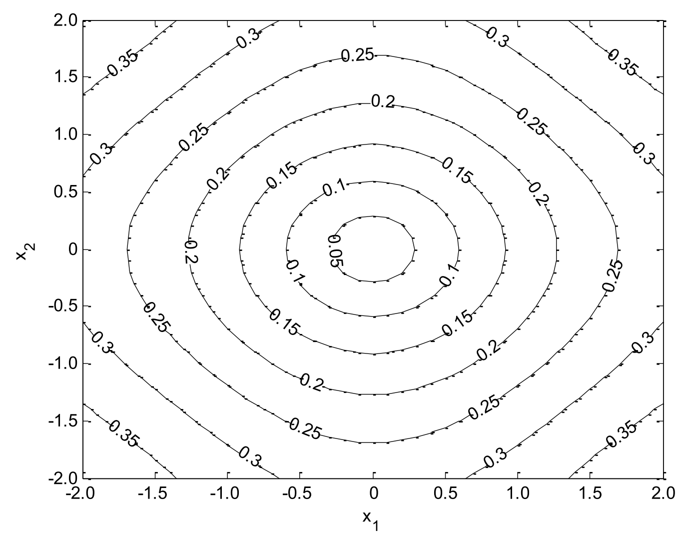
2.3. Correntropy

The correntropy between two arbitrary random variables X and Y is defined as follows [24]

$$V_\sigma(X, Y) = E\left[\kappa_\sigma(X - Y)\right] \tag{5}$$



(a) Kernel size $\sigma = 1.0$



(b) Kernel size $\sigma = 1.5$

Fig. 1. Contours of CIM in 2D sample space.

$$\kappa_\sigma(\bullet) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\bullet)^2}{2\sigma^2}\right) \tag{6}$$

where $\kappa_\sigma(\bullet)$ is the Gaussian kernel function, σ is the kernel size, and $E[\bullet]$ denotes the mathematical expectation. In practice, the correntropy for a finite number of data $\boldsymbol{x} = [x_1, \cdots, x_N]^{\mathrm{T}}$ and $\boldsymbol{y} = [y_1, \cdots, y_N]^{\mathrm{T}}$ is estimated as follows

$$\hat{V}_\sigma(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{N} \sum_{i=1}^{N} \kappa_\sigma(x_i - y_i) \tag{7}$$

Correntropy induces a metric, referred to as the correntropy induced metric (CIM), in the sample space. Fig. 1 shows the contours of CIM measured distance from point (x_1, x_2) to the origin in the 2D sample space. We find that the CIM behaves like an L2 norm when the point is close to the origin, an L1 norm when the point is apart from the origin, and an L0 norm when the points is further apart from the origin. So the correntropy is inherently robust to outliers. We also find that, compared with a small kernel size, the CIM with a larger kernel size has a larger L2 norm region and a tighter L1 norm region. It means that the kernel size controls the scale of CIM.