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Optimal design of continuous time irrational filter with a set of fractional order gammatone components via norm relaxed sequential quadratic programming approach

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ABSTRACT

This paper proposes a continuous time irrational filter structure via a set of the fractional order Gammatone components instead of via a set of integer order Gammatone components. The filter design problem is formulated as a nonsmooth and nonconvex infinite constrained optimization problem. The nonsmooth function is approximated by a smooth operator. The domain of the constraint functions is sampled into a set of finite discrete points so the infinite constrained optimization problem is approximated by a finite constrained optimization problem. To find a near globally optimal solution, the norm relaxed sequential quadratic programming approach is applied to find the locally optimal solutions of this nonconvex optimization problem. The current or the previous locally optimal solutions are kicked out by adding the random vectors to them. The locally optimal solutions with the lower objective functional values are retained and the locally optimal solutions with the higher objective functional values are discarded. By iterating the above procedures, a near globally optimal solution is found. The designed filter is applied to perform the denoising. It is found that the signal to noise ratio of the designed filter is higher than those of the filters designed by the conventional gradient descent approach and the genetic algorithm method, while the required computational power of our proposed method is lower than those of the conventional gradient descent approach and the genetic algorithm method. Also, the signal to noise ratio of the filter with the fractional order Gammatone components is higher than those of the filter with the integer order Gammatone components and the conventional rational infinite impulse response filters.

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1. Introduction

Continuous time infinite impulse response filters are widely employed in many engineering and science applications such as in an analogue to digital conversion system [1] and an audio system [2–7]. The most common continuous time infinite impulse response filter is a continuous time rational filter. If the poles of a continuous time rational filter are located on the left hand side of the complex plane, then the stability of the filter is guaranteed [8]. However, the frequency response of a continuous time rational filter is expressed as the ratio of two complex valued polynomial functions of frequency [8]. The overall response is only linearly related to its numerator response, but it is highly nonlinearly related to its denominator response. Therefore, the rational filter is very difficult to achieve the best performance. Also, the rational filter may suffer from an ill posed problem. Hence, the rational filter may not be the best structure for practical applications.

To address this issue, a continuous time irrational filter is employed. As there is no dominator in the frequency response of the irrational filter, the ill posed problem is avoided. Moreover, if the filter structure is properly chosen, then the continuous time irrational filter could achieve a better performance compared to the rational filter. However, the stability of the continuous time irrational filter is not guaranteed. To ensure the stability, the absolute integral of the impulse response of the continuous time irrational filter is required to be computed [8]. Nevertheless, the absolute integral of the impulse response of the continuous time irrational filter may be difficult to be evaluated.

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To address this issue, a continuous time integer order Gammatone filter is proposed [2–7]. The impulse response of the continuous time integer order Gammatone filter is expressed as the product of a polynomial function, an exponential decay function and a cosine function [2–7]. It is worth noting that the impulse response of the continuous time integer order Gammatone filter is bounded by its envelope. Here, the envelope is the absolute value of the product of the polynomial function and the exponential decay function. Also, the absolute integral of the product of a high order monomial function and the exponential decay function can be computed via an iterative approach. Therefore, a bound for the absolute integral of the impulse response of the continuous time integer order Gammatone filter can be deduced via the triangular inequality theorem. This guarantees the stability of the continuous time integer order Gammatone filter. Besides, for a time function in the form of the product of a polynomial function and an exponential decay function, its frequency response is the weighted sum of the high order derivative of the continuous time Fourier transform of the exponential decay function. Also, for a signal in the form of the product of a time function and a cosine function, its frequency response is the modulation of the continuous time Fourier transform of the time function. Hence, the frequency response of the continuous time integer order Gammatone filter is mainly determined by the frequency response of its exponential decay function. As the continuous time Fourier transform of the exponential decay function is localized in the frequency domain (here, the localization in the frequency domain refers to the extent of the filter that most of the energy of the signal is within a certain frequency band), the passband of the continuous time integer order Gammatone filter is very narrow. Therefore, the continuous time integer order Gammatone filter can achieve a very good performance particularly for an application required a very narrow passband. As analyzing audio signals requires a very narrow band filter [2–7], the continuous time integer order Gammatone filter is widely used in many audio applications. However, the continuous time integer order Gammatone filter cannot achieve a good performance for an application required a wide passband. This is because the total number of design variables is very limited.

To address this issue, a filter with a set of integer order Gammatone components is employed. Here, the center frequencies of different integer order Gammatone components are different so that the bandwidth of the filter corresponding to the weight sum of the integer order Gammatone components is larger than that corresponding to only one integer order Gammatone component. However, as the order of the power function of the integer order Gammatone component is integer valued while the order of the exponential decay function as well as the frequency, the amplitude and the phase of the cosine function are real valued, the design problem is actually a hybrid optimization problem (here, the hybrid optimization problem refers to the optimization problem with some decision variables are in a continuous set and some decision variables are in a discrete set). Nevertheless, it is very difficult to find the solution of this kind of optimization problems. Up to the authors' understanding, there is no existing result on finding the solution of the hybrid optimization formulated based on the optimal design of the filter with the set of integer order Gammatone components [9]. As the Gammatone components have not been optimally designed, the performance of the filter with the integer order Gammtone components are quite poor. Therefore, the signal to noise ratio of the filter with the set of integer order Gammatone components is very low.

To address this difficulty, this paper proposes the filter structure with a set of fractional order Gammatone components. Here, the fractional order Gammatone components refer to those Gammatone components with the power functions having the fractional orders. As the integration of the product of a power function and another time function can be interpreted as the moment of the time function, the integration of the impulse response of the filter with the fractional order Gammatone components can be interpreted as the weighted sum of the fractional order moments of the frequency modulated exponential decay pulses. On the other hand, the integration of the impulse response of the filter with the integer order Gammatone components are the weighted sum of the integer order moments of the frequency modulated exponential decay pulses. As the set of integers is a subset of the fractional numbers, in general the filter with the fractional order Gammatone components should achieve better performances compared to the filter with the integer order Gammatone filters. Besides, from the optimization viewpoint, as all the decision variables of the optimization problem corresponding to the design of the filter with the fractional order Gammatone components are in a continuous set, the design problem is a real valued optimization problem. In this case, the difficulty for finding the solution of the hybrid optimization does not occur. However, as the frequency responses of this type of filters involve the fractional derivative [10], the magnitude responses of this type of filters are expressed as the highly nonconvex functions of the component parameters [11]. The required computational powers for finding the near globally optimal solutions of this type of optimization problems [12] are very high. Up to the authors' understanding, there is no existing result on the design of this type of filters.

The major contributions and novelties of this paper are to propose a new structure and a new methodology for designing a continuous time irrational filter. From the structure viewpoint, the filter composes of fractional order Gammatone components. In this case, the integration of the impulse response of the filter can be interpreted as the weight sum of the fractional order moments of the frequency modulated exponential decay pulses. Compared to the filter with the integer order Gammatone components where the integration of its impulse response is the weight sum of the integer order moments of the frequency modulated exponential decay pulses, the filter with the fractional order Gammatone components can represent a wider class of filters. This is because the set of integers is a subset of the fractional numbers. Therefore, the filter designed based on the fractional order Gammatone components outperforms that based on the integer order Gammatone components.

From the design viewpoint, it is important to reduce the required computational powers for finding the near globally optimal solutions of the optimization problems. Therefore, this paper employs an absolute operator instead of using the modulus square operator for formulating the objective function. In this case, the order of the objective function is reduced. As a result, the required computational power for finding the near globally optimal solution of the optimization problem is reduced. However, the optimization problem is nonsmooth. Hence, the conventional gradient descent approach cannot be applied for finding the locally optimal solutions of the optimization problem. To address this difficulty, the nonsmooth operator in the objective function is approximated by a smooth operator so that a norm relaxed sequential quadratic programming based method [13–15] can be applied for finding the locally optimal solutions of the optimization problem efficiently via a conventional gradient descent approach. Here, the sequential quadratic programming based method assumes that the initial guess of the decision vector of the optimization problem is in its feasible set. Also, both the objective function and the constraint functions are smooth. This method is to find the directions of descents via solving a sequence of quadratic programming problems. A set of initial guesses of the decision vectors is constructed based on the obtained directions of descents. Here, the obtained initial guesses of the decision vectors are guaranteed to be in the feasible set of the original optimization problem. Also, the objective func-

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