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Accuracy analysis of complex sinusoid amplitude and phase estimation by means of the interpolated discrete-time Fourier transform algorithm

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ABSTRACT

In this paper the accuracy of the complex sinusoid amplitude and phase estimators provided by the Interpolated Weighted Discrete-Time Fourier Transform (IpWDTFT) algorithm is analyzed. It is shown that the use of the WDTFT spectral sample corresponding to the estimated sinusoid frequency allows to minimize the estimator asymptotic Mean Square Error (MSE) due to additive wideband noise. Also, the expression of the asymptotic expected sum-squared fitting error in the reconstruction of the complex sinusoid by means of the IpWDTFT algorithm is derived. All the derived expressions are verified through computer simulations.

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1. Introduction

In many engineering fields, such as communications, instrumentation, radar, sonar, and vibration analysis, complex or real noisy sinusoids are employed, and their parameters need to be accurately estimated in real time. Since in practice it is quite difficult to ensure coherent sampling of the acquired waveform, interpolated Fourier algorithms are often used [1–15]. They estimate the sinusoid frequency using a two-step search procedure. In the first step (called coarse-search) the peak of the discrete signal spectrum is identified. Then, the second step (called fine-search) is aimed at compensating the picket-fence effect on the estimated sinusoid parameters by estimating the inter-bin location of the signal frequency through the interpolation of either the module [1–12] or the complex value [9–15] of Discrete Fourier Transform (DFT) samples in the neighbour of the spectrum peak. Then the sinusoid amplitude and phase are estimated by using a DFT sample located closely to the obtained frequency estimate. To reduce the frequency estimator bias more than two interpolation points are used [16–22] at the cost of an increased estimation variance [19]. To reduce the effect of the spectral interference due to undesired tones, such as the fundamental image component, harmonics or interharmonics, acquired data are weighted by a suitable window function. The Maximum Sidelobe Decay (MSD) cosine windows [23], known also as the Rife–Vincent class I windows [1], are often employed since spectrum interpolation can be performed using very simple formulas [1–6,17–22]. Moreover, these windows strongly reduce the spectral interference from distant tones since their spectrum sidelobes have the highest decay rate among all the cosine windows with a given number of terms H [23].

However, to the best of the authors' knowledge, the identification of the Discrete Time Fourier Transform (DTFT) sample that allows to minimize the contribution of wideband noise on the amplitude and phase estimates returned by the interpolation algorithm has not been yet analysed in the scientific literature. This is the aim of this paper. Specifically, the DTFT sample ensuring amplitude and phase estimators with minimum asymptotic Mean Square Error (MSE) is identified in the case of a complex noisy sinusoid weighted by a MSD window. The paper is organized as follows. In Section 2 the general expression for the Interpolated Weighted Discrete Time Fourier Transform (IpWDTFT) amplitude and phase estimators are provided. In Section 3 the expressions of the asymptotic MSEs for both considered estimators are derived and the spectral sample that allows to minimize the estimators MSE is identified. Moreover, to evaluate the accuracy of the reconstruction of the complex sinusoid provided by the IpWDTFT algorithm, the expression of the related asymptotic expected sum-

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squared fitting error is also derived. All the derived expressions are then verified by means of computer simulations in Section 4. Finally, some conclusions are drawn in Section 5.

2. The IpWDTFT amplitude and phase estimators

The analyzed complex discrete-time noisy sinusoid is expressed as:

$$x(m) = Ae^{j(2\pi \frac{f_{in}}{f_s} m+\phi)} + e(m) = s(m) + e(m),$$
$$m = 0, 1, 2, \dots, M-1 \quad (1)$$

where f_{in}, A, and ϕ, are the frequency, the amplitude, and the phase of the continuous-time complex sinusoid $s(m)$, f_s is the sampling rate, $e(\cdot)$ is a complex additive white Gaussian noise with zero mean and variance σ^2, and M is the number of acquired samples. The ratio between the sinusoid frequency and the sampling rate can be expressed as:

$$\frac{f_{in}}{f_s} = \frac{\nu}{M} = \frac{l+\delta}{M}, \quad (2)$$

where $\nu = l + \delta$ is the number of acquired sinusoid cycles or the normalized frequency expressed in bins, in which l is the rounded value of ν and δ $(-0.5 \le \delta < 0.5)$ is the inter-bin location of the signal frequency. For coherent sampling (that is $\delta = 0$) the signal frequency is located exactly in a Discrete Fourier Transform (DFT) bin and the error due to the picket-fence effect is null [3]. Conversely, non-coherent sampling (that is $\delta \neq 0$) usually occurs in practice and the compensation of the picket-fence effect on the estimated sinusoid parameters need the estimation of the inter-bin frequency location δ. This represents the goal of the so called interpolated Fourier algorithms.

The integer part of the number of acquired sinusoid cycles, l, can be exactly determined through a maximum search procedure applied to the periodogram of the analyzed signal when the related frequency Signal-to-Noise Ratio (*SNR*) is higher than the threshold [24]. In this case, the estimators for ν and δ exhibit the same statistical proprieties.

To reduce the contribution from other disturbance tones such as harmonics or inter-harmonics, the signal $x(m)$ is weighted by a suitable window $w(\cdot)$ so obtaining the weighted signal $x_w(m) = x(m) \cdot w(m)$.

The DTFT of the windowed signal $x_w(m)$, called Weighted DTFT (WDTFT) in the following, is given by:

$$X_w(\lambda) \triangleq \sum_{m=0}^{M-1} x(m)e^{-j2\pi \frac{\lambda}{M} m}$$
$$= AW(\lambda - \nu)e^{j\phi} + E_w(\lambda), \qquad \lambda \in [0, M) \quad (3)$$

where $W(\cdot)$ and $E_w(\cdot)$ are the DTFT of the considered window $w(\cdot)$ and of the weighted wideband noise $e_w(m) = w(m) \cdot e(m)$, respectively.

A H-term MSD window $(H \ge 2)$ is adopted in the following. It is expressed by [23,25]:

$$w(m) = \sum_{h=0}^{H-1} (-1)^h a_h \cos\left(2\pi \frac{h}{M} m\right), \quad m = 0, 1, \dots, M-1 \quad (4)$$

where a_h, $h = 0, \dots, H-1$ are the window coefficients, given by [5]: $a_0 = C_{2H-2}^{H-1}/2^{2H-2}$ and $a_h = C_{2H-2}^{H-h-1}/2^{2H-3}$, $h = 1, 2, \dots, H-1$, where $C_m^p = p!/[(p-q)!q!]$. It is worth noticing that (4) represents the rectangular window, i.e. to the absence of weighting, when $H = 1$.

For $|\lambda| \ll M$, the DTFT of (4) can be approximated with high accuracy by [5]:

$$W(\lambda) = \frac{M \sin(\pi\lambda)}{2^{2H-2}\pi\lambda} \frac{(2H-2)!}{\prod_{h=1}^{H-1}(h^2 - \lambda^2)} e^{-j\pi \frac{M-1}{M}\lambda}. \quad (5)$$

Interesting figures of merit of a cosine window are: the Equivalent Noise BandWidth (*ENBW*), the Normalized Peak Signal Gain (*NPSG*), the Normalized Noise Power Gain (*NNPG*), and the Scalloping Loss (*SL*) [25]. For the H-term MSD window $(H \ge 2)$ they have the following expressions [5]:

$$ENBW \triangleq \frac{NNPG}{(NPSG)^2} = \frac{C_{4H-4}^{2H-2}}{(C_{2H-2}^{H-1})^2}, \quad (6)$$

$$NPSG \triangleq \frac{1}{M}\sum_{m=0}^{M-1} w(m) = \frac{W(0)}{M} = a_0 = \frac{C_{2H-2}^{H-1}}{2^{2H-2}}, \quad (7)$$

$$NNPG \triangleq \frac{1}{M}\sum_{m=0}^{M-1} w^2(m) = a_0^2 + 0.5\sum_{h=1}^{H-1} a_h^2 = \frac{C_{4H-4}^{2H-2}}{2^{4H-4}}, \quad (8)$$

$$SL(\delta) \triangleq \frac{|W(\delta)|}{W(0)} = \frac{\sin(\pi\delta)}{\pi\delta} \frac{[(H-1)!]^2}{\prod_{h=1}^{H-1}(h^2-\delta^2)}. \quad (9)$$

From (9) it follows that the minimum and the maximum values of $SL(\delta)$ are reached at $\delta = -0.5$ and $\delta = 0$, respectively. Moreover, $SNR \triangleq A^2/\sigma^2$ represents the Signal-to-Noise Ratio of the analyzed sinusoid.

In the following the sinusoid parameters are estimated through an IpWDTFT algorithm. As in the classical IpDFT algorithm [1], the inter-bin frequency location δ is firstly estimated by interpolating the spectral samples located around the discrete spectrum peak, i.e. $X_w(l+k)$, $k = 0, \pm 1, \pm 2, \dots$. In order to minimize the required computational effort, the amplitude and the phase estimators proposed in the literature are often based on the DFT sample corresponding to the discrete spectrum peak, i.e. $X_w(l)$, which is already available [1–3,5]. Conversely, the IpWDTFT algorithm considers the WDTFT sample related to the estimated frequency, that is $X_w(l+\hat{\delta})$, to achieve, as we will see in the following, the minimum sensitivity to wideband noise.

In the following the bias and the variance of the used IpWDTFT frequency estimator $\hat{\delta}$ are denoted as $b_{\hat{\delta}}$ and $\sigma_{\hat{\delta}}^2$, respectively.

By neglecting the contribution of wideband noise, and since the frequency estimation error $\Delta\delta = \hat{\delta} - \delta$ assumes negligible values, evaluating (3) at $\lambda = l + \hat{\delta} + r$, we obtain:

$$X_w(l+\hat{\delta}+r) \cong AW(r)e^{j\phi}, \quad -0.5 \le r < 0.5 \quad (10)$$

where the term r represents the deviation from the estimated normalized frequency expressed in bins (off-frequency term) $\hat{\nu}$. It is worth noticing that, while the inter-bin frequency location δ represents the fractional part of the sinusoid normalized frequency ν, and it is related to frequencies f_{in} and f_s through (2), r is simply a displacement parameter belonging to the range $[-0.5, 0.5)$, which is completely unrelated to any signal or acquisition system parameters. It has been introduced in (10) in order to determine which WDTFT sample minimizes the contribution of wideband noise to the amplitude and phase IpWDTFT estimators.

Expression (10) suggests to consider the following amplitude and phase estimators:

$$\hat{A}_r = \frac{|X_w(l+\hat{\delta}+r)|}{|W(r)|}, \quad -0.5 \le r < 0.5 \quad (11)$$

and

$$\hat{\phi}_r = angle\{X_w(l+\hat{\delta}+r)\} + \pi\frac{M-1}{M}r - angle\{W_0(r)\},$$
$$-0.5 \le r < 0.5, \quad (12)$$

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