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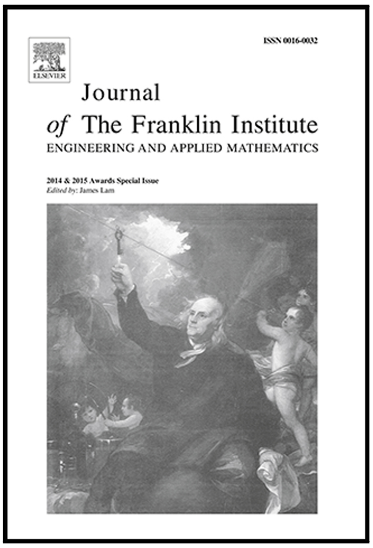

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Critically Damped Stabilization of Inverted-Pendulum Systems Using Continuous-Time Cascade Linear Model Predictive Control

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Abstract
In this paper, a cascade continuous-time linear MPC controller with 7 well defined parameters is proposed to stabilize the inverted pendulum system around the desired equilibrium points. The proposed controller scheme is composed of two MPC controllers in cascade. These controllers are tuned so as to obtain a doubly critically damped behavior for the inner and outer loops. Explicit generalized expressions of the MPC controller parameters in relation to the inverted pendulum characteristics and the chosen horizon time parameter with the robustness analysis of the controlled system are given to facilitate the controller design for a wide class of inverted pendulum systems. Simulation results show that the MPC controller is compared favorably to the well known parallel two-PID control scheme.

Keywords: Inverted pendulum, Single Input-Multi Output (SIMO) nonlinear system, Model predictive control (MPC), critically damped stabilization, proportional-integral-derivative (PID) control, Cascade control scheme.

Nomenclature:

$x_1 = \theta$: pendulum angle	a_i, b_i : linearized model parameters	ω_{n0}, ω_{n1} : natural frequencies
$x_2 = \dot{\theta}$: pendulum angle velocity	A_i, B_i, E_i, G_i : model Prediction parameters	ω_u : stability limit parameter
x_3 : cart position	K_{1i}, K_{2i} : controller gains	ρ_1, ρ_2, μ : weight factors
x_4 : cart velocity	c, d, e : normalized controller gains	h_1, h_2 : horizon times
$\hat{x}_i(t)$: predicted state at time t	c_0, d_0, e_0 : normalized controller gains with critically damped constraint	J, J_1, J_2 : cost functions
m : pendulum mass	ε : positive design parameter	x_{1d} : desirable angle position
M : cart mass	$x_{10}(t), x_{20}(t)$: constrained state variables	x_{3d} : desirable cart position
l : pendulum length	$S(s)$: sensitivity function	$\alpha = l\varepsilon^{-1}$
g : gravity acceleration	$C(s)$: complementary sensitivity function	$\beta = g\varepsilon^{-1}$
u, u_0 : control variables	P : noise power	σ : standard deviation of noise
u_{0n} : normalized control variable		

1. INTRODUCTION

The inverted pendulum is one of the most important studied systems in academic and industrial researches. This system can be used to approximate many complex models such as rockets during liftoff, bipedal walking, cranes, and robots. There are several versions for inverted pendulum systems, among them the most familiar ones are the rotational single-arm pendulum [1], the cart inverted pendulum [2], and the double inverted pendulum [3]. Some less common versions are the rotational two-link pendulum [4] and the triple inverted pendulum [5].The inverted pendulum is nonlinear,