Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 000 (2017) 1–19
www.elsevier.com/locate/jfranklin

A new idea on almost sure permanence and uniform boundedness for a stochastic predator–prey model

Xiaoling Zou, Jingliang Lv*

Department of Mathematics, Harbin Institute of Technology, Weihai 264209, PR China

Received 23 September 2016; received in revised form 5 May 2017; accepted 2 July 2017
Available online xxx

Abstract

The existence and uniqueness of stationary distribution and ergodic properties of a stochastic system are obtained. Especially, different from the existing methods, a new method is introduced to analyze almost sure permanence and uniform boundedness of the stochastic predator–prey model. This new idea is based on geometric structure of invariant set for a stochastic system. More specifically, we obtain our main conclusions by showing the invariant set for the stochastic population system lies in the interior of the first quadrant. It is interesting and surprising that the stochastic population model can guarantee a uniform boundedness almost surely. Some numerical simulations are carried out to support our results.

1. Introduction

The hiding behavior of prey on the dynamics of predator–prey interactions is one of the major issues in applied mathematics and theoretical ecology. It is well known that traditional ways of prey refuge incorporating in predator–prey interactions are to consider two types of refuge in the literature: those that protect a constant fraction of prey and those that protect a constant number of prey [1]. A predator–prey model with Holling type II functional response

* Corresponding author.
E-mail address: ljl3188@163.com (J. Lv).

http://dx.doi.org/10.1016/j.jfranklin.2017.07.012

incorporating a constant number of prey refuge reads:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha\left(1 - \frac{x}{K}\right)x - \frac{\beta(x-m)y}{1+a(x-m)},$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = -dy + \frac{c\beta(x-m)y}{1+a(x-m)}. \tag{1.1}$$

Chen et al. [2] showed instability, global stability properties of equilibria, existence and uniqueness of limit cycle for model (1.1). Ji and Wu gave qualitative analysis of a predator–prey model with constant-rate prey harvesting incorporating a constant prey refuge in literature [3]. In model (1.1), authors only considered intra-species competition for prey ($-\alpha x^2/K$) but not for predator. In a real world, the certain environment confines that the predator needs density regulation. And the results with density regulation of predator are different from the results without density regulation of predator, therefore, in the ecological significance view, we should consider the density regulation of predator [4,5]. By virtue of experiments, the findings imply that ecological theory needs to take realistic levels of predator dependence into account [6]. Many scholars have considered both prey and predator intra-species competitions in ecology by mathematical models, such as [7–9]. So when we take the intra-species competitions for both predator and prey into consideration ($-\alpha x^2/K$ and $-dy^2/L$), the corresponding model with a constant proportion of prey refuge becomes:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha\left(1 - \frac{x}{K}\right)x - \frac{\beta(1-m)xy}{1+a(1-m)x},$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = d\left(-1 - \frac{y}{L}\right)y + \frac{c\beta(1-m)xy}{1+a(1-m)x}, \tag{1.2}$$

where $m \in [0, 1)$ is a constant and this leaves $(1-m)x$ of the prey available to the predator.

In fact, an important ecosystem component is that population dynamics is inevitably affected by environmental noises (see e.g. Gard [10,11]). Deterministic models have some limitations in mathematical modeling of ecological systems, such as they are quite difficult to fit data perfectly and to predict the future dynamics of the system accurately [12]. May has pointed out that parameters in systems exhibited random fluctuations to a greater or lesser extent due to environmental noises [13]. Here we consider the environmental fluctuations in population dynamics (see e.g. [8,9,14–16] and references cited therein) and have the following stochastic system:

$$\mathrm{d}x = \left[\alpha\left(1 - \frac{x}{K}\right)x - \frac{\beta(1-m)xy}{1+a(1-m)x}\right]\mathrm{d}t + \sigma_1 x\mathrm{d}B_1(t), \ \ x(0) > 0,$$
$$\mathrm{d}y = \left[d\left(-1 - \frac{y}{L}\right)y + \frac{c\beta(1-m)xy}{1+a(1-m)x}\right]\mathrm{d}t + \sigma_2 y\mathrm{d}B_2(t), \ \ y(0) > 0, \tag{1.3}$$

where $B_1(t)$, $B_2(t)$ are mutually independent *Brownian* motions, σ_1^2 and σ_2^2 represent intensity of white noises.

For a population system, permanence is a very important and interesting property, which means that the population system will survive forever. Generally speaking, the existing definitions of permanence to stochastic population systems are as follows: stochastic permanence [17], persistence in mean [18,19], almost sure permanence [20]. Many results have been obtained on all types of permanence that we mentioned above. However, the main method is the Lyapunov method that has been applied to consider the existing permanence of stochastic

Please cite this article as: X. Zou, J. Lv, A new idea on almost sure permanence and uniform boundedness for a stochastic predator–prey model, Journal of the Franklin Institute (2017), http://dx.doi.org/10.1016/j.jfranklin.2017.07.012