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ScienceDirect
Journal of the Franklin Institute 000 (2017) 1–24
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Local exponential stabilization via boundary feedback controllers for a class of unstable semi-linear parabolic distributed parameter processes

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Received 4 November 2015; received in revised form 30 April 2017; accepted 29 May 2017
Available online xxx

Abstract

This paper addresses the problem of local exponential stabilization via boundary feedback controllers for a class of nonlinear distributed parameter processes described by a scalar semi-linear parabolic partial differential equation (PDE). Both the domain-averaged measurement form and the boundary measurement form are considered. For the boundary measurement form, the collocated boundary measurement case and the non-collocated boundary measurement case are studied, respectively. For both domain-averaged measurement case and collocated boundary measurement case, a static output feedback controller is constructed. An observer-based output feedback controller is constructed for the non-collocated boundary measurement case. It is shown by the contraction semigroup theory and the Lyapunov's direct method that the resulting closed-loop system has a unique classical solution and is locally exponentially stable under sufficient conditions given in term of linear matrix inequalities (LMIs). The estimation of domain of attraction is also discussed for the resulting closed-loop system in this paper. Finally, the effectiveness of the proposed control methods is illustrated by a numerical example.

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http://dx.doi.org/10.1016/j.jfranklin.2017.05.044
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1. Introduction

Almost all physical processes are spatially distributed in nature so that their behavior is related to both spatial position and time, for example, thermal diffusion processes, flexible structure systems and chemical engineering, etc. [1–5]. The mathematical models describing these processes are typically derived from the dynamic conservation equations and take the form of partial differential equations (PDEs). The parabolic PDEs describes the spatiotemporal dynamics of diffusion-convection-reaction processes, such as rapid thermal processing, plasma reactors, crystal growth processes, to name a few [4]. Due to that the suggested controllers are easy and convenient for real implementation, moreover, feedback control design via a finite number of actuators/sensors for the PDE systems is of practical importance. In general, control actions in the PDE systems are either distributed over the entire (or part thereof) spatial domain (distributed controls), applied at the boundary (or part thereof) of the spatial domain (boundary controls), or active only at specified points of this domain (pointwise controls) [6]. Among them, boundary control is generally considered to be more realistic in many applications, because it requires relatively few actuators in comparison to distributed control.

Therefore, the investigation of boundary control design for parabolic PDE systems has attracted a lot of attention in the past decades [7–13]. By combining the finite-difference method and the finite-dimensional backstepping method, the problem of boundary control design has been addressed in [7–9] for a class of linear/nonlinear reaction-diffusion systems. The idea utilized in [7–9] is that the finite-difference method is initially applied to the PDE model to derive an approximate model consisting of a set of ordinary differential equations (ODEs) in time. The resulting ODE model is subsequently used as the basis of the synthesis of finite-dimensional backstepping boundary control design. However, the order of the resulting ODE system used for the control design in [7–9] may be very large for yielding the desired approximation degree, leading to complex controller design and high dimensionality of the resulting controllers. Motivated by the fact that the dominant dynamics of parabolic PDE systems can be approximately described by a low-dimensional ODE system, predictive boundary control was proposed for linear parabolic PDE systems [10], sampled-data boundary control design was developed in [11] for a linear parabolic PDE with non-collocated observation, robust boundary control design was presented in [12] for semi-linear parabolic PDE systems, and adaptive neural boundary control design was developed in [13] for semi-linear parabolic PDE systems, in which Galerkin's method was applied to derive a low-dimensional ODE model used as the basis of finite-dimensional control design. A potential drawback of results [10–13] is the inherent loss of process information due to the truncation before the controller design.

To obtain a better control performance and a higher control precision, a PDE-based backstepping boundary feedback control design has been developed in [14–16] for linear parabolic PDE systems. The results in [14–16] have been extended to address adaptive boundary control [17], tracking boundary control [18], and sliding-mode output feedback boundary control [19] of linear parabolic PDE systems. More recently, boundary control and observer design [15] have been extended to address boundary exponential stabilization of a quasi-linear 2×2 PDE system [20], extended Luenberger observer design for semi-linear PDE systems [21], boundary control of coupled reaction-diffusion processes [22], and output regulation of boundary controlled parabolic PDEs [23], respectively. Some boundary feedback control design methods different from the backstepping boundary control design in [14–23] have been developed in [24] for a linear heat process with unbounded matched perturbation and