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Routh-type table test for zero distribution of polynomials with commensurate fractional and integer degrees☆

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Abstract

This paper mainly presents Routh-type table test methods for zero distribution of polynomials with commensurate fractional degrees on the left-half plane, right-half plane and imaginary axis in the complex plane. The proposed tabular methods are derived for extension and generalization of the Routh test, which is widely used in controls for zero distribution of polynomials with integer degrees. Singular cases are discussed and handled efficiently and simply. Necessary and sufficient conditions for the second singular case are completely analyzed in terms of symmetric zeros. A particular property is revealed that a polynomial with commensurate fractional degrees without pure imaginary zero may still be stable in the presence of the second singular case, which is impossible for a real polynomial with integer degrees. Furthermore, we present a test to solve the zero distribution problem with respect to general sector region for polynomials with commensurate fractional degrees and real/complex coefficients. Finally, numerical examples are given to illustrate the correctness and effectiveness of the results. The proposed methods have broad application areas, including various systems, circuits and control.

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1. Introduction

Determining zero distribution of a real or complex polynomial is a mathematical problem with a long history and various solutions, and is very important for stability and performance analysis in control and systems areas. Among that, the celebrated Routh table test [1] (also see [2], pp. 177–185) is the most efficient method for determining the distribution of polynomial zeros in the left and right half complex planes as well as the imaginary axis. Furthermore, since Routh-type results/techniques have more implications and applications than the original polynomial zero distribution analysis, various research topics and results associated with the Routh test have been developed especially by control, circuit and system theorists. To mention a few, Ho et al. [3] generalized the Hermite-Biehler theorem to prove the Routh table and revealed a property to check whether a zero moves across the imaginary axis or not from the neighbor rows. That property was also obtained through a simple proof [4]. Lev-Ari et al. [5] proposed fast triangular factorization to evaluate the inertia of Bezoutian matrices with displacement structure, which leads to the Routh-Hurwitz and Schur-Cohn tests as well as more general test for regions with arbitrary circles and straight lines. Due to the extensive works, Routh test has been extended to solve zero distribution problems for complex polynomials (e.g., [5] and [6]) and subregions enclosed by various type of boundaries (e.g., which are transformed from imaginary axis by any rational function [7], or arbitrary lines, or segments [8]). Recently, Bistritz [9] presented fraction-free forms of the Routh test for complex and real integer polynomials.

Technical development of Routh test also includes the treatment for singular cases. For the first singular case, the so-called epsilon-replacement method is widely known but requires careful treatment for the presence of some imaginary axis zeros [2,10]. Some computationally efficient methods are proposed via left-shifting the corresponding row to eliminate those leading zero elements [8]. Furthermore, Pal and Kailath [11] provided a quite complete solution for all singular cases by analyzing the quasi-Hankel Bezoutions. Genin [12] presented a different and effective test by using the quotient polynomials obtained from the Euclid division, whereby the first singular case was incorporated into the regular non-singular cases. Recently, Choghadi and Talebi [13] proposed an additional check on the distance of the consecutive zero rows for the multiplicities of pure imaginary zeros when the second singular case occurred.

It is noticed that in addition to stability test, the normal Routh test also has wide applications such as to H_2/H_∞ norm computation and model reduction [14], analysis of linear analog circuits [15], and many industrial examples [16].

Since the past decade, fractional order systems have attracted increasing attention from the mathematics, control, circuit and industrial community due to its physical existence and potential improvement to systems [17–26]. For instance, real capacitors (with different kinds of dielectrics) and coils (with inherent eddy current and hysteresis losses) are experimentally verified to have response characteristics of fractional orders [21,22]. Recently, the existence of solutions to impulsive nonlinear fractional order systems and impulsive fractional differential inclusions has been investigated in [25] and [26], respectively.

One of the most fundamental problems in the field of fractional order systems is the stability analysis. Matignon theorem [27] and its generalization [28] indicate that a linear time invariant fractional order system is BIBO stable if and only if its characteristic function as a polynomial with fractional degrees (PFD) has no closed-RHP zero. Thereafter, the stability and stabilization of fractional order systems have been extensively investigated, e.g., by LMI-based methods [29,30], root locus method [31], analysis for rational order systems [32], and a review on various systems including linear or nonlinear, distributed, and time delay [33]. The classical Routh-Hurwitz criterion