Available online at www.sciencedirect.com

ScienceDirect

Journal of the Franklin Institute 354 (2017) 332–344

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Stability of Markov jump systems with quadratic terms and its application to RLC circuits ☆

Alessandro N. Vargas[a,*], Gisela Pujol[b], Leonardo Acho[b]

[a]*Universidade Tecnológica Federal do Paraná, UTFPR, Av. Alberto Carazzai 1640, 86300-000 Cornelio Procópio-PR, Brazil*

[b]*CoDAlab (Control, Dynamics and Applications), Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Comte d'Urgell, 187, 08036 Barcelona, Spain*

Received 27 July 2015; received in revised form 25 July 2016; accepted 26 August 2016
Available online 5 October 2016

Abstract

The paper presents results for the second moment stability of continuous-time Markov jump systems with quadratic terms, aiming for engineering applications. Quadratic terms stem from physical constraints in applications, as in electronic circuits based on resistor (R), inductor (L), and capacitor (C). In the paper, an RLC circuit supplied a load driven by jumps produced by a Markov chain—the RLC circuit used sensors that measured the quadratic of electrical currents and voltages. Our result was then used to design a stabilizing controller for the RLC circuit with measurements based on that quadratic terms. The experimental data confirm the usefulness of our approach.

1. Introduction

Systems subject to Markovian jumps have received attention in recent years because of their potential for representing processes subject to abrupt variations—see, for instance, some recent

☆Research supported in part by the Spanish Ministry of Economy and Competitiveness through the research projects DPI2015-64170-R/MINECO/FEDER, DPI2011-25822, DPI2015-64170-R/MINECO/FEDER; by the Government of Catalonia (Spain) through 2014SGR859; and by the Brazilian agencies FAPESP Grants 03/06736-7; CNPq Grant 304856/2007-0; and CAPES Grant Programa PVE 88881.030423/2013-01.
*Corresponding author.
E-mail address: avargas@utfpr.edu.br (A.N. Vargas).

http://dx.doi.org/10.1016/j.jfranklin.2016.08.031

applications in economics [6,16], robotics [21], and direct current (DC) motors [17,19,20,25]. In the linear context, recent contributions for Markov jump systems can be found in the monographs [3,7] and in the papers [5,9,18,22,26–29]; for the nonlinear counterpart, contributions can be found in [14,23,24,32], just to cite a few.

Although characterizing the stability of nonlinear Markov jump systems has been a topic of intensive research [14,23,24,31], little attention has been paid to the stability of quadratic Markov jump systems. In reality, to the best of the authors' knowledge, this paper is the first to consider quadratic terms for Markov jump systems. Presenting easy-to-check conditions to guarantee the stability of such systems represents the main contribution of this paper.

To clarify our findings, we now formalize the quadratic Markov jump system under study. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ be a fixed, filtered probability space governing the following Itô stochastic differential equation with Markov jumps:

$$\mathrm{d}x(t) = A_{\theta(t)}x(t)\mathrm{d}t + \begin{bmatrix} x(t)'G_{1,\theta(t)}x(t) \\ \vdots \\ x(t)'G_{n,\theta(t)}x(t) \end{bmatrix} \mathrm{d}t + H_{\theta(t)}\mathrm{d}w(t),$$

$$\forall t \geq 0, \; x(0) = x_0 \in \mathbb{R}^n, \tag{1}$$

where $x(t)$ denotes an n-dimensional system state, $w(t)$ denotes a standard r-dimensional Brownian motion, and $\{\theta(t)\}$ represents an irreducible continuous-time Markov process having $\mathcal{S} = \{1, \ldots, N\}$ as state space. As usual, $x(t)$, $w(t)$, and $\theta(t)$ are mutually independent random variables at $t \geq 0$. The value of each tuple of matrices $(A_i, H_i, G_{1,i}, \ldots, G_{n,i})$, $i = 1, \ldots, N$, is given.

The main contribution of this paper is to present conditions to assure that the quadratic Markov jump system in Eq. (1) is second moment stable, as follows.

Definition 1.1. ([1, Definition 11.3.1, p. 188]). We say that the quadratic Markov jump system in Eq. (1) is second moment stable if there exists some constant $c = c(x_0)$ such that

$$\mathrm{E}[\|x(t)\|^2] \leq c, \quad \forall t \geq 0.$$

Now, consider the elements of the n-dimensional vector $x(t)$ written explicitly in the form $x(t) \equiv [x_{[1]}(t), \ldots, x_{[n]}(t)]'$.

Assumption 1.1. The elements $x_{[\ell]}(t)$, $\ell = 1, \ldots, n$, are uniformly bounded from below almost surely. As a result, there exist values $\mu_1, \ldots, \mu_n$ such that

$$\mu_\ell \leq \liminf_{t \to \infty} x_{[\ell]}(t), \quad \ell = 1, \ldots, n, \tag{2}$$

almost surely.

The condition in Assumption 1.1 is fundamental in our approach. Assumption 1.1 states a lower bound for $x_{[\ell]}(t)$, but an upper bound on $x_{[\ell]}(t)$ may not exist, that is, $x_{[\ell]}(t)$ could diverge to infinity as t goes to infinity. In order to prevent such divergent behaviour in Eq. (1), we present conditions to guarantee the second moment stability, as in Definition 1.1.

The assumption that $x_{[\ell]}(t)$ has a lower bound is mild, since there are many applications for which the system states are bounded from below. For instance, in DC motors, both the angular velocity and the electrical current are bounded from below [19,20].