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Bipartite synchronization in a network of nonlinear systems: A contraction approach

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Abstract

This paper studies the bipartite synchronization in a network of nonlinear systems with collaborative and antagonistic interactions. Under the assumption that the signed graph is structurally balanced and the considered domain does not contain the origin, we use contraction theory to obtain some sufficient conditions such that the network admits a bipartite synchronization solution. These conditions are described by coupling matrices and the contractivity of lower-dimensional dynamic systems. In particular, if the nonlinear system satisfies a one-sided Lipschitz condition and the coupling matrices are identical, we also obtain some sufficient conditions about the second smallest eigenvalue of signed graph for the bipartite synchronization. Some numerical examples are presented to illustrate the effectiveness of the obtained results.

1. Introduction

The last decade has witnessed considerable research effort to achieve consensus (synchronization) in a network of systems, such as consensus of multi-agent systems [1–4], synchronization of complex networks [5–12] etc. In order to achieve consensus collaborative relationship among agents is a common assumption in most of the literature. However, competition (antagonists) is another inherent relationship among agents in natural and engineering systems, such as competing

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species and competitive cellular neurons [13–15], social networks [16,17], and personalized recommendation [18,19]. It is reasonable to ask what collective behavior will appear in a network when there coexist both collaborative and competitive interactions. Consequently, some fascinating collective behaviors appear (e.g. bipartite consensus [20], multi-consensus [21]). Bipartite consensus was considered in some recently published papers [20,22–27].

Altafini first introduced the bipartite consensus concept [20], where bipartite consensus means that all agents converge to a value which is the same for all in modulus but not in sign. When there coexist both collaborative and competitive relationships in a network, a signed graph consisting of positive and negative edges can be used to describe relationships among agents, that is, positive and negative edges present collaborative and competitive relationships, respectively. In paper [20], Altafini studied the collective behavior in a network of integrators over signed graph, and proved that the network admits a bipartite consensus solution if and only if the signed graph is structurally balanced. This result has been extended to general linear multi-agent systems [23,27] and dynamic output feedback control [24], where each node is modeled by a linear time-invariant (LTI) system. Under a weak connectivity assumption that the signed network has a spanning tree, the papers [25,26] obtained some sufficient conditions for bipartite consensus of multi-integrator systems on directed signed networks. In addition, the authors of [22] studied the bipartite flock control problem in multi-integrator systems. The above papers just deal with the case that each agent is modeled by an integrator or a linear time-invariant system. However, almost all real systems are nonlinear. To the best of authors' knowledge, the problem of bipartite synchronization in a network of nonlinear systems has not been investigated in the existing literature.

Motivated by the above observations, this paper studies the bipartite synchronization in a network of nonlinear systems using contraction approach. The contribution of this paper is two-fold. First, when the inner coupling matrices are different and the signed graph is structurally balanced, we obtain some sufficient conditions such that the network admits a bipartite synchronization solution. These conditions are described by coupling matrices and the contractivity of lower-dimensional dynamic systems. Second, we investigate the special case that the inner coupling matrices are identical, and the nonlinear system satisfies a one-sided Lipschitz condition. When the signed graph is structurally balanced and at least one eigenvalue of the linearization about the origin has a positive real part, we obtain some simple conditions to guarantee a bipartite synchronization solution. These simple conditions are about the second smallest eigenvalue of signed graph and can be easily checked. Finally, we show some numerical examples to illustrate the obtained results.

The paper is organized as follows. Section 2 presents problem statement. Section 3 investigates the bipartite synchronization in a network of nonlinear systems. Section 4 shows some numerical examples to verify the obtained results. Section 5 summarizes our conclusions and describes future work. Appendix A presents some definitions and facts about signed graphs. Appendix B recalls some facts about contraction theory. Appendix C shows some definitions and notations about dynamic systems.

2. Problem statement

The following notations are used throughout this article. The notation $|x|$ denotes the Euclidean norm of vector x. $||A||$ denotes the induced matrix norm of A corresponding to the vector norm $|\cdot|$. The notation I_N denotes the N-dimensional identity matrix. Let $\mathbf{0}_N(\mathbf{1}_N)$ be the N-