

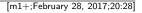
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Modeling of discrete-time fractional-order state space systems using the balanced truncation method

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Abstract

This paper presents a new approach to approximation of linear time-invariant (LTI) discrete-time fractional-order state space SISO systems by means of the SVD-originated balanced truncation (BT) method applied to an FIR-based representation of the fractional-order system. This specific representation of the system enables to introduce simple, analytical formulas for determination of the Cholesky factor-izations of the controllability and observability Gramians, which contributes to significant improvement of the computational efficiency of the BT method. As a model reduction result for the fractional-order systems we obtain a low-order rational (integer-order) state space system. Simulation experiments show a high efficiency of the introduced methodology both in terms of the approximation accuracy of the model and low time complexity of the approximation algorithm.

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Keywords: Fractional-order system; Discrete-time system; Model order reduction; Balanced truncation method.

1. Introduction

Fractional-order dynamic systems have attracted a considerable research interest. It is the specific properties of fractional-order models that make them more adequate in modeling of selected (not only) industrial systems. However, the discrete-time Grünwald–Letnikov fractional-order difference (FD), to be considered here, may lead to computational explosion. Therefore,

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a number of various concepts have been developed to approximate, or model discrete-time fractional difference systems. Those concepts are mainly based on two types of applications, where 1) approximators are used to model the discrete-time fractional-order difference involved in a fractional-order system and 2) the whole fractional-order system is modeled by a rational, integer-order approximator.

In the first case, the solutions have led to e.g. least-squares (LS) fit of an impulse/step response of a discrete-time integer-order IIR filter [3,5,29]. On the other hand, a fit of a FIR filter to FD has been analyzed in the frequency and time domains [11,12,25], also in terms of time-varying filters [23]. An alternative approach has been the employment of an approximating filter incorporating discrete-time Laguerre functions [24]. In this case, Laguerre-based equivalents to the fractional-order difference, called Laguerre-based difference, have been proposed [24]. The approximation of the fractional-order difference is obtained as a finite-length implementation of the Laguerre-based difference, called finite Laguerre-based difference. Another approach to use the Laguerre filters has been proposed in [14].

In the second case, there are a number of methods for approximation of fractional order systems by integer, high order approximators [9,20,21]. One of the conceptually simplest, but computationally involving, approximation methods for fractional-order systems is the involvement of both approximation of the fractional-order difference and system performance in a high-order state space model, by use of the so-called 'expanded' state equation [6,15]. In this case, the fractional-order difference is calculated using the finite-length implementation of the Grünwald–Letnikov difference. In [22] it has been shown that the expanded state equation (BT) method, which however suffers from computational burden. A similar application of model order reduction methods for fractional-order continuous-time systems under the Oustaloup approximation of the fractional-order derivative has been presented in [13].

This paper presents a new method for approximation of discrete-time fractional-order state space systems using the balanced truncation applied to a specific, FIR-based representation of a fractional-order system. The paper is organized as follows. Having introduced the approximation problem for discrete-time fractional-order systems in Section 1, a representation method for discrete-time fractional-order state space systems is presented in Section 2. Section 3 recalls the model order reduction problem for discrete-time state space systems via the Balanced Truncation method. The main result in terms of a simple, analytical solution method for the Gramian factorizations used in the BT algorithm is presented in Section 4. A simulation example of Section 5 confirms the effectiveness of the introduced methodology both in terms of high modeling accuracy and low time complexity of the approximation algorithm. Conclusions of Section 6 complete the paper.

2. System representation

Consider a commensurate fractional-order discrete-time state space single input single output (SISO) system

$$\Delta^{\alpha} x(t+1) = Ax(t) + Bu(t), \quad x_0$$

$$y(t) = Cx(t) + Du(t)$$
(1)

where $t = 0, 1, ..., x(t) \in \mathbb{R}^n$ is the state vector, u(t) and y(t) are input and output signals, respectively, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$, $C \in \mathbb{R}^{1 \times n}$ and $D \in \mathbb{R}$.

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