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Modeling of discrete-time fractional-order state space systems using the balanced truncation method

Rafał Stanisławski*, Marek Rydel, Krzysztof J. Latawiec

Department of Electrical, Control and Computer Engineering, Opole University of Technology, ul. Prószkowska 76, 45-758 Opole, Opole, Poland

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Abstract

This paper presents a new approach to approximation of linear time-invariant (LTI) discrete-time fractional-order state space SISO systems by means of the SVD-originated balanced truncation (BT) method applied to an FIR-based representation of the fractional-order system. This specific representation of the system enables to introduce simple, analytical formulas for determination of the Cholesky factorizations of the controllability and observability Gramians, which contributes to significant improvement of the computational efficiency of the BT method. As a model reduction result for the fractional-order systems we obtain a low-order rational (integer-order) state space system. Simulation experiments show a high efficiency of the introduced methodology both in terms of the approximation accuracy of the model and low time complexity of the approximation algorithm.

Keywords: Fractional-order system; Discrete-time system; Model order reduction; Balanced truncation method.

1. Introduction

Fractional-order dynamic systems have attracted a considerable research interest. It is the specific properties of fractional-order models that make them more adequate in modeling of selected (not only) industrial systems. However, the discrete-time Grünwald–Letnikov fractional-order difference (FD), to be considered here, may lead to computational explosion. Therefore,

* Corresponding author.
E-mail addresses: r.stanislawski@po.opole.pl (R. Stanisławski), m.rydel@po.opole.pl (M. Rydel), k.latawiec@po.opole.pl (K.J. Latawiec).

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a number of various concepts have been developed to approximate, or model discrete-time fractional difference systems. Those concepts are mainly based on two types of applications, where 1) approximators are used to model the discrete-time fractional-order difference involved in a fractional-order system and 2) the whole fractional-order system is modeled by a rational, integer-order approximator.

In the first case, the solutions have led to e.g. least-squares (LS) fit of an impulse/step response of a discrete-time integer-order IIR filter [3,5,29]. On the other hand, a fit of a FIR filter to FD has been analyzed in the frequency and time domains [11,12,25], also in terms of time-varying filters [23]. An alternative approach has been the employment of an approximating filter incorporating discrete-time Laguerre functions [24]. In this case, Laguerre-based equivalents to the fractional-order difference, called Laguerre-based difference, have been proposed [24]. The approximation of the fractional-order difference is obtained as a finite-length implementation of the Laguerre-based difference, called finite Laguerre-based difference. Another approach to use the Laguerre filters has been proposed in [14].

In the second case, there are a number of methods for approximation of fractional order systems by integer, high order approximators [9,20,21]. One of the conceptually simplest, but computationally involving, approximation methods for fractional-order systems is the involvement of both approximation of the fractional-order difference and system performance in a high-order state space model, by use of the so-called 'expanded' state equation [6,15]. In this case, the fractional-order difference is calculated using the finite-length implementation of the Grünwald–Letnikov difference. In [22] it has been shown that the expanded state equation model can be effectively reduced to a low-order approximator using the balanced truncation (BT) method, which however suffers from computational burden. A similar application of model order reduction methods for fractional-order continuous-time systems under the Oustaloup approximation of the fractional-order derivative has been presented in [13].

This paper presents a new method for approximation of discrete-time fractional-order state space systems using the balanced truncation applied to a specific, FIR-based representation of a fractional-order system. The paper is organized as follows. Having introduced the approximation problem for discrete-time fractional-order systems in Section 1, a representation method for discrete-time fractional-order state space systems is presented in Section 2. Section 3 recalls the model order reduction problem for discrete-time state space systems via the Balanced Truncation method. The main result in terms of a simple, analytical solution method for the Gramian factorizations used in the BT algorithm is presented in Section 4. A simulation example of Section 5 confirms the effectiveness of the introduced methodology both in terms of high modeling accuracy and low time complexity of the approximation algorithm. Conclusions of Section 6 complete the paper.

2. System representation

Consider a commensurate fractional-order discrete-time state space single input single output (SISO) system

$$\begin{aligned} \Delta^{\alpha} x(t+1) &= Ax(t) + Bu(t), \quad x_0 \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

where $t = 0, 1, \ldots,$ $x(t) \in \Re^n$ is the state vector, $u(t)$ and $y(t)$ are input and output signals, respectively, $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$, $C \in \Re^{1\times n}$ and $D \in \Re$.