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Output formation-containment analysis and design for general linear time-invariant multi-agent systems ☆

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Abstract

Output formation-containment control problems for general linear time-invariant multi-agent systems with directed topologies are dealt with. Output formation-containment means that the outputs of leaders achieve the predefined formation, and at the same time the outputs of followers converge to the convex hull formed by the outputs of leaders. Firstly, static output protocols are presented for leaders and followers respectively. Then output formation-containment problems of multi-agent systems are transformed into asymptotic stability problems. Sufficient conditions with less computation complexity are proposed for multi-agent systems to achieve the output formation-containment. An explicit expression for the time-varying output formation reference function is derived to describe the macroscopic movement of the whole output formation-containment. Explicit expressions to describe the relationship among the outputs of followers, the time-varying output formation for the leaders and the output formation reference are derived. It is proven that the outputs of followers not only converge to the convex hull formed by those of leaders but also achieve certain time-varying formation specified by the convex combination of the desired output formation for the leaders. Moreover, an approach to determine the gain matrices in the protocols is given for multi-agent systems to achieve the output formation-containment. Finally, numerical simulations are provided to demonstrate the effectiveness of the theoretical results.

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1. Introduction

Over the past decade, the research on multi-agent systems and their collective behavior has been one of the major issues in control engineering due to their variety of applications such as formation control of mobile robots [1,2], cooperative control of unmanned aerial vehicles [3,4], and communication among sensor networks [5,6]. In particular, consensus problem has received significant attention. The objective of consensus is to make all agents reach an agreement on certain variables of interest. In recent years, consensus problems for first-order, second-order, general linear time-invariant (LTI) and nonlinear multi-agent systems have been studied extensively [7–17].

The above-mentioned work on consensus mainly focused on leaderless coordination of multi-agent systems. However, in real applications, there might exist one or even multiple leaders in a multi-agent system. Consensus with one leader is often named as consensus tracking. In [18], distributed consensus tracking algorithms were proposed for first-order multi-agent systems with directed interaction topologies. Consensus tracking problems for second-order multi-agent systems with switching interaction topologies were studied in [19]. Ni and Cheng [20] addressed consensus tracking problems for general LTI multi-agent systems. For general LTI multi-agent systems to achieve consensus tracking, Li et al. [21] investigated the case that the control input of the leader is nonzero and not available to the followers. In the presence of multiple leaders, containment problem which requires that the states/outputs of followers converge to the convex hull formed by the states/outputs of leaders arises. Ji et al. [22] discussed containment problems using a hybrid stop-go control strategy. Meng et al. [23] proposed a model-independent control law for multi-agent systems with rigid bodies to achieve finite-time containment. For first-order multi-agent systems, Notarstefano et al. [24] showed that by using a linear consensus protocol, the containment can be achieved as long as the switching undirected interaction topology is jointly connected. Necessary and sufficient conditions for first-order and second-order multi-agent systems to achieve containment were derived by Liu et al. [25]. Containment problems for second-order multi-agent systems with time-varying delays were investigated in [26].

The dynamics of each agent is restricted to be of low order in [22–26]. Containment problems for general LTI multi-agent systems with directed interaction topologies were considered by Li et al. [27]. Dong et al. [28] proposed sufficient conditions for general LTI singular multi-agent systems with time delays to achieve containment. For general LTI multi-agent systems, Liu et al. [29] classified the agents into boundary agents and internal agents, and presented necessary and sufficient conditions for the states of internal agents to converge to a convex combination of the states of the boundary agents. Necessary and sufficient conditions for general LTI multi-agent systems to achieve output containment were presented in [30]. Mittag-Leffler stability methods were applied to prove that the containment can be achieved by the fractional-order multi-agent systems in [31]. An aperiodic sampled-data based containment protocol was constructed using neighboring information with uncertainly time-varying sampling intervals for general LTI multi-agent systems in [32]. However, in [27–32], it was assumed that there exists no coordination among leaders.

In some practical applications, leaders may need to coordinate with each other to accomplish certain complicated tasks, such as formation control. The problems that the states/outputs of leaders converge to desired formations and the states/outputs of followers converge to the convex hull formed by the states/outputs of leaders are regarded as state/output formation-containment problems. Formation-containment control can find its applications in formation attack of multiple unmanned/manned combat aerial vehicles (CAVs), cooperative transportation of multi-robot