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Controllability and dissipativity analysis for linear systems with derivative input ☆

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Abstract

This paper investigates controllability and dissipativity analysis problem for linear systems with derivative input. First, we show that linear systems with derivative input can be changed into special class of singular systems. A necessary and sufficient condition for the controllability of linear systems with derivative input is derived. Second, a necessary and sufficient condition for the dissipativity of this type of linear systems with quadratic form storage function is derived using the linear matrix inequality approach. Based on this condition, the dissipativity of singular systems with impulse behavior is investigated and a parametrization for all possible solutions on dissipativity is presented. Finally, two examples are given to show the validity of the derived results.

1. Introduction

Based on an input–output relationship, dissipativity theory provides a framework for the design and analysis of control systems from energy point of view. Its essence is that there exists a nonnegative energy function (storage function), such that the system energy consumption is less

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than the energy supply [1,2]. Dissipativity theory has been proved to be an effective method in robot control [5,7] and adaptive control [4,6] , nonlinear H_∞ control [3] and so on.

So far, some progress on linear systems with derivative input has been achieved. For example, some necessary and sufficient conditions for the state controllability of multivariable time-invariant linear systems with input-derivative control are derived in [8]. The problem of finding inputs which generate zero outputs for linear systems with the derivative of input is investigated in [9]. In [10], a method is developed to find input vectors which generate zero output vectors for linear systems with state equation containing any finite number of derivatives of the input vector. In [11], the state equation with derivative input preliminarily has been explored and in [12], the quadratic index optimal control problem of linear systems with derivative input has been considered. In [16], the dissipative and passive state feedback controller for linear systems with derivative input are studied recently.

On the other hand, there has been a growing research interest of singular systems in control community, since such systems can provide accurate representations for interconnected large-scale systems, economic systems, electrical networks, power systems, and mechanical systems [14,15,18,19]. Meanwhile, dissipative control for singular systems has attracted particular interest for its various applications. In [20], some necessary and sufficient conditions are given respectively to make continuous/discrete singular systems admissible and strictly dissipative. In [21], linear matrix inequalities are used to characterize necessary and sufficient conditions for dissipativity of singular systems. Based on results in [21], design of output feedback controllers to achieve dissipativity of the closed-loop systems is investigated in [22]. In [23], the problem of delay-dependent α-dissipativity is investigated for continuous singular systems with time-delay. The positive realness and dissipativity of singular systems are investigated by the image space in [13,17], respectively.

In this paper, we first show how to change singular systems to the linear systems with derivative input. Then, a necessary and sufficient condition for the controllability of linear systems with derivative input is given. As a special case, this controllability condition includes the controllability condition of singular systems. Second, a necessary and sufficient condition for dissipativity of systems with derivative input is derived in terms of LMIs. It is known that impulsive behavior is the main feature of singular systems, which is different from normal systems. However, most of existing results so far on dissipativity for singular systems are only for impulse-free systems due to the difficulty of characterization of the relationship between dissipativity and impulsive behavior. Motivated by this observation, this paper proposes a method of investigating the dissipativity of general singular systems with impulsive behavior by transforming it into linear systems with derivative input. Third, the dissipativity for singular systems with impulsive behavior is investigated and all possible solutions are derived in a parametrization form. This result is valid regardless of the impulsive behavior, which is significantly different from the existing results. Finally two practical examples are given to demonstrate the effectiveness of the results in this paper.

Notations: Let $\mathbf{R}$ and $\mathbf{R}^+$ denote the sets of real and nonnegative real numbers respectively. $\mathbf{R}^{m\times n}$ denotes the set of $m\times n$ matrices with real elements. $\mathbf{C}_p^{h-1}$ denotes $h-1$-times piecewise continuously differentiable functions. Let P be a square matrix, and P is said to be symmetric if $P=P^{\mathrm{T}}$. We say that P (necessarily symmetric) is positive semi-definite if $v^{\mathrm{T}}Pv\geq 0$ for all vectors v. We write $P\geq 0$ if P is positive semi-definite. Negative semi-definiteness can be defined in a similar fashion. The notation deg($*$) is the degree of a polynomial. For a symmetric