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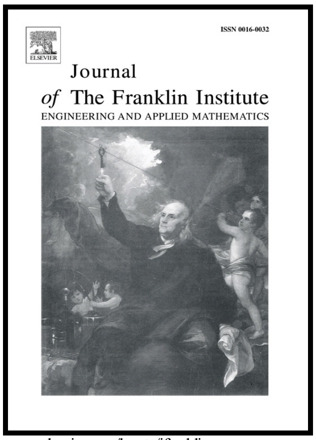

A finite iterative algorithm for solving the complex generalized coupled Sylvester matrix equations by using the linear operators

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A finite iterative algorithm for solving the complex generalized coupled Sylvester matrix equations by using the linear operators ☆

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Abstract

By introducing the real linear operator, this paper offers a finite iterative algorithm for solving the complex generalized coupled Sylvester matrix equations. The properties of this algorithm are discussed and the finite convergence of this algorithm is proven. This algorithm unifies some finite iterative algorithms proposed by the earlier papers. Two numerical examples are offered to illustrate the effectiveness of the proposed algorithm.

Keywords: Finite iterative algorithm, Complex generalized coupled matrix equations, Real inner product, Linear operator
Mathematical sub classification numbers: 15Axx Basic linear algebra; 65Fxx Numerical linear algebra

1. Introduction

Matrix equations often arise from system theory [1, 2, 3, 4], control theory [5, 6] and stability analysis [7]. How to solve these matrix equations become an important topic which have received much attention [8, 9, 10, 11, 12]. Some different methods for solving matrix equations were established.

Recently, the hierarchical identification principle was used to solve the matrix equations by Ding and Chen [13, 14, 15, 16]. By combining the hierarchical identification principle and the gradient iterative method of the simple matrix equations, the gradient-based iterative algorithms were established for $\boldsymbol{AXB}+\boldsymbol{CXD}=\boldsymbol{F}$, $\boldsymbol{AXB}+\boldsymbol{CX}^{\mathrm{T}}\boldsymbol{D}=\boldsymbol{F}$ [17, 18] and for the coupled matrix equations $\boldsymbol{A}_1\boldsymbol{XB}_1=\boldsymbol{F}_1$ and $\boldsymbol{A}_2\boldsymbol{XB}_2=\boldsymbol{F}_2$ [19]. For more references, one can refer to [20, 21, 22, 23, 24]. To improve the previous results, Zhou developed the weighted least squares iterative algorithms for solving the general coupled matrix equations in [25, 26].

Another important method for solving the matrix equations is the finite iterative method [27]. Originated from the conjugate gradient method and experienced two periods [28], this method has become very popular. The finite iterative algorithms have been established for solving real (complex) matrix equation and coupled matrix equations. For examples, by constructing the orthogonal residual matrices, the skew-symmetric solution of matrix equation $\boldsymbol{AXB}=\boldsymbol{C}$ was established [29]. By introducing the real inner product, Wu et al. [30] suggested a finite iterative algorithm for solving the extended Sylvester-conjugate matrix equation $\boldsymbol{AXB}+\boldsymbol{C}\bar{\boldsymbol{X}}\boldsymbol{D}=\boldsymbol{F}$. To obtain the reflexive and anti-reflexive solutions, a finite iterative algorithms for solving the matrix equation $\boldsymbol{A}_1\boldsymbol{X}_1\boldsymbol{B}_1+\boldsymbol{A}_2\boldsymbol{X}_2\boldsymbol{B}_2=\boldsymbol{C}$ was established in [31]. The following matrix equation

$$\sum_{i=1}^{p}\boldsymbol{A}_i\boldsymbol{X}\boldsymbol{B}_i+\sum_{i=1}^{q}\boldsymbol{C}_i\bar{\boldsymbol{X}}\boldsymbol{D}_i+\sum_{i=1}^{r}\boldsymbol{G}_i\boldsymbol{X}^T\boldsymbol{H}_i+\sum_{i=1}^{s}\boldsymbol{M}_i\boldsymbol{X}^H\boldsymbol{N}_i=\boldsymbol{F} \quad (1)$$

was discussed and the finite iterative algorithm was established [32].

Some finite iterative algorithms for solving the coupled matrix equations were established. For examples, a finite iterative algorithm for solving the coupled matrix equations $\boldsymbol{AYB}=\boldsymbol{E},\boldsymbol{CYD}=\boldsymbol{F}$ over generalized

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