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Short communication

Improved synthesis method for Markov jump systems with uncertain transition rates ☆

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Abstract

This paper focuses on exploring a new controller design method for Markov jump linear systems with uncertain transition rates. By fully considering the property of transition rates and the characteristic of uncertain domains, a more tailored technique instead of the traditional Young inequality is proposed to bind the uncertain terms. Benefited from the new technique, an improved method for controller design is obtained, which involves much less decision variables and has no any compromise on conservatism comparing with the existing ones. A numerical example is given to show the validity of the developed results.

1. Introduction

For Markov jump systems (MJSs), the transition rates in the jump process affect the system behavior to a great extent. Unfortunately, despite the transition rates are so important, it is in general difficult to obtain their precise values. Therefore, the investigation for MJS with uncertain transition rates has attracted researchers' attention over the past decade (see, e.g. [1–12] and the references therein).

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In the literature, three different types of descriptions about uncertain transition rates were considered in the context of robust stabilization. The first one is the polytopic description where the transition rate matrix is assumed to be in a convex hull with known vertices [1,2]. The second one assumes that the elements in the transition rate matrix are partly unknown [3–6]. The third one is described in an element-wise way [7–12]. In this case, the elements of the mode transition rate matrix are measured in practice and error bounds are given at the same time. In this paper, we consider the element-wise uncertainty in the transition rate.

In [7–9] the controller design is considered for MJSs with element-wise description for uncertain transition rates but the relationship among the transition rates is neglected, which leads to the conservatism. In [10,11], by considering the relationship among the transition rates, low conservative results are proposed. However, the controller design methods in [10,11] are in terms of a class of nonlinear matrix inequalities (NLMIs), which cannot still be *completely* solved up to now [13]. In [12] an appealing method is proposed for controller design, which not only maintains the merit of low conservatism like those of [10,11] but also only involves pure LMIs instead of NLMIs. Although some encouraging progresses are made for the controller design method of MJSs with uncertain transition rates, there is further room for investigation. For example, all of the existing low conservative methods [10–12] use the usual Young inequality to bind the uncertain terms, which lead to more decision variable to be introduced. It is known that on the one hand, more decision variables imply more complex and time-consuming application; but on the other hand, the reduction of decision variables easily leads to the increase of conservatism [14]. Therefore, developing a new method that involves less decision variable while has no any compromise on conservatism will be a significant and challenging work, which motivates the present study.

In this paper, the problems of analysis and synthesis are revisited for MJSs with uncertain transition rates. By fully considering the properties of the transition rates and the characteristic of the uncertain domains, a more tailored technique instead of Young inequality is proposed to bind the uncertain terms. Benefited from this method, the obtained stability criterion involves much less decision variables while has no any compromise on conservatism comparing with the existing ones. Then, based on the stability criterion, a new LMI method for controller design is presented, which possesses the same merit as the proposed stability criterion. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Notation: In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript T denotes transpose. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). For the notation $(\Omega, \mathcal{F}, \mathbb{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathbb{P}$ is the probability measure on $\mathbb{F}$. We use $*$ as an ellipsis for the terms that are introduced by symmetry. $\mathrm{diag}\{\cdot, \cdots, \cdot\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Given the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and consider the following MJS:

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t), \quad t \geq 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^{n_u}$ is the control input. For the sake of simplicity, a matrix $M(r(t) = i)$ will be denoted by M_i. A_i and B_i are known constant matrices. The mode