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Full- and reduced-order observer design for rectangular descriptor systems with unknown inputs ☆

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Abstract

In this paper, methods are proposed to design Luenberger type full- and reduced-order observers for rectangular descriptor systems with unknown inputs. These methods are based on the effect of pre- and post-multiplicative operation of a linear transformation, derived here by means of simple matrix theory. Sufficient conditions for the existence of observers are given and proved. Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

In the last few decades, many researchers have given a lot of attention on the analysis and design of descriptor systems as these are general enough to provide a solid understanding for the inner dynamics of underlying physical systems [1–3]. Many physical systems can be modeled as the system of differential algebraic equations and can be written in the form of descriptor systems, but it is not always necessary that number of variables of interest and equations are the same. Thus, we are here concerned with rectangular systems. Some real life applications are electrical circuits [4], chemical control processes [5], constrained mechanics [6], and biological systems [7] to name a few.

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An observer is a mathematical realization which uses the input and output informations of a given system and its output asymptotically approaches the true state values of the given system. Observer design problem for normal systems has received a great attention in the literature [8–14] and the techniques used for them have been extended successfully to descriptor linear [15–22] and nonlinear [23–26] systems. There are many practical situations where control systems arise with noise or disturbances, so the problem of observer design for systems with unknown inputs is of great importance. Descriptor systems are very sensitive to slight input changes, because differentiation terms of inputs exist in their solution [1]. Thus, observers design problem for descriptor systems with unknown inputs is more important than observer design for normal systems with unknown inputs. Many researchers have worked either on observer design of square descriptor system with unknown inputs or on rectangular descriptor system with unknown inputs only in dynamical part [27–32], but results on rectangular descriptor systems with unknown inputs in dynamical as well as in measurement equation are limited [33,34]. Considering unknown inputs in measurement equation is very important since, unlike the known inputs, unknown inputs cannot be eliminated from the measurement equation without loss of generality. Concepts of generalized Sylvester equation and generalized inverse have been used for the design of observers for descriptor systems with unknown inputs [33]. Koenig [34] presents a method to design proportional multiple-integral observer for descriptor systems for estimating simultaneously the states, faults, and unknown inputs, but the order of observer is greater than the number of states in the given system. Ting et al. [35] have designed observer for normal system with unknown inputs by transforming it into a descriptor system through a series of linear transformations in the state and output equations.

It has been shown that the condition of detectability is necessary for the existence of Luenberger type observers for descriptor systems [2]. In this paper, our assumptions on the system operators are imitated from the papers [33,34] as these conditions are very less restrictive for the design of observers for systems having unknown inputs. As compared to these articles, the proposed method is straightforward and simple to understand and implement. Our approach is based on the restricted system equivalent theory and does not require the concept of generalized Sylvester equation. In this note, one full column rank matrix R is designed in such a way that its pre-multiplication to some matrices gives the design approach for full-order observer and its post-multiplication reveals the reduced-order observer design approach. The order of the proposed reduced-order observer is less than the dynamical order of the given descriptor system.

Rest of the paper is organized as follows. In Section 2, descriptor system with unknown inputs in dynamical and measurement equations is transformed to descriptor system with unknown inputs only in dynamical part by using the known results. Section 3 presents full-order observer design approach. Based on the results of Sections 2 and 3, reduced-order observer is designed in Section 4. To illustrate the derived results, numerical examples are given in Section 5. Finally, Section 6 concludes the paper.

2. Preliminaries

Consider the following linear time invariant descriptor system with unknown inputs:

$$E^*\dot{x} = A^*x + B^*u + F^*v, \tag{1a}$$

$$y^* = C^*x + G^*v, \tag{1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, $v \in \mathbb{R}^q$, and $y^* \in \mathbb{R}^p$ are the state vector, the control input vector, the unknown input vector, and the output vector, respectively. $E^* \in \mathbb{R}^{m\times n}$, $A^* \in \mathbb{R}^{m\times n}$, $B^* \in \mathbb{R}^{m\times k}$,