Available online at www.sciencedirect.com

ScienceDirect

Journal of the Franklin Institute 352 (2015) 5106–5121

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Globally asymptotic stabilization of stochastic nonlinear systems in strict-feedback form ☆

Shuangshuang Xiong[a,b], Quanxin Zhu[a], Feng Jiang[c,*]

[a] *School of Mathematical Sciences and Institute of Finance and Statistics, Nanjing Normal University, Nanjing, 210023 Jiangsu, China*

[b] *Advanced Control Systems Laboratory, School of Electronics and Information Engineering, Beijing Jiaotong University, Beijing 100044, China*

[c] *School of Statistics and Mathematics, Zhongnan University of Economics and Law, Wuhan, 430073 Hubei, China*

Received 27 December 2014; received in revised form 24 July 2015; accepted 19 August 2015

Available online 7 September 2015

Abstract

In this paper, we study the problem of globally asymptotic stabilization for a class of stochastic nonlinear systems. Without requiring the condition that the coefficients are smooth functions, this paper is a first try to apply the backstepping control design method to prove the globally asymptotic stability in probability of stochastic nonlinear systems in strict-feedback form. A continuous control law is constructed by a novel systematic design algorithm. Finally, we provide an example to show the effectiveness of the theoretical results.

1. Introduction

As is well known, stochastic differential equations play a more and more important role in many fields such as economic and finance, physics, mechanics, electric and control engineering. The stability analysis is one of the most important works in the research of stochastic differential

☆This work was jointly supported by the National Natural Science Foundation of China (61374080, 61304067), the Natural Science Foundation of Hubei Province of China (2013CFB443), and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

*Corresponding author.

E-mail address: fjiang78@163.com (F. Jiang).

http://dx.doi.org/10.1016/j.jfranklin.2015.08.013

equations. As a consequence, there have appeared a large number of stability results in the literature. For instance, see the books [1–7] and the references therein.

It is clear that the classical and powerful technique applied in the stability analysis for stochastic differential equations has been mainly dominated by the Lyapunov function approach, see, for example [3,5,6,8,9]. For example, Deng and Krstić [10] extended the result on inverse optimal stabilization for general systems with deterministic uncertainties to the stochastic case by developing a simple algorithmic design for a class of strict-feedback systems. Stochastic versions of the LaSalle theorem were proved in [11,12], and Mao [13] further generalized the stochastic LaSalle theorem without using the local Lipschitz condition by the requirement that the system has a unique strong solution. Ignatyev and Mandrekar [14] established stochastic Barbashin–Krasovskii theorem. Rami and Ghaoui [15] systematically considered stochastic stability properties of jump linear systems and the relationship among various moment and sample path stability properties. Zhang and Chen [16] applied the spectrum-based technique to investigate the asymptotical mean-square stability of linear stochastic time-invariant systems. By using the spectral technique, Feng et al. studied the problem of exponential stabilization for continuous-time stochastic systems in [17]. Recently, Hu et al. [18] proved the stability and boundedness of nonlinear hybrid stochastic differential delay equations by using different types of Lyapunov functions. Based on the Lyapunov–Krasovskii functional, Razumikhin technique with a stochastic version and the linear matrix inequalities technique, Zhu and Song [19] obtained some novel sufficient stability conditions for a class of impulsive nonlinear stochastic differential equations with mixed time delays. In [20], Huang studied the stabilization and destabilization of stochastic nonlinear differential equations. Du et al. [21] obtained a new sufficient condition for stability in distribution of stochastic differential delay equations with Markovian switching. By designing discrete-time feedback controls, Mao [22] investigated the mean-square exponential stabilization of continuous-time hybrid stochastic differential equations.

In this paper, we consider the following nonlinear stochastic differential equations with the strict-feedback systems driven by white noise:

$$\begin{aligned} dx_i &= x_{i+1}\,dt + g_i^T(\bar{x}_i)\,dw, \quad i=1,\ldots,n-1,\\ dx_n &= (f(x)+u)\,dt + g_n^T(\bar{x}_n)\,dw, \end{aligned} \tag{1.1}$$

where $(\Omega, \mathcal{F}, \mathrm{P})$ is a probability space and $\mathcal{F}_t$ is a filtration of sub-σ-fields of $\mathcal{F}$, $x{:=}(x_1,\ldots,x_n)^T = \{\bar{x}_n(t), \mathcal{F}_t; 0 \le t < \infty\}$ is a continuous, adapted $\mathbb{R}^n$-valued measurable process, $w=\{w_t, \mathcal{F}_t; 0 \le t < \infty\}$ is an m-dimensional Brownian motion, $u \in \mathbb{R}$ represents the control input of the system. Let $\bar{x}_i = (x_1,\ldots,x_i)^T$, the functions $f(x): \mathbb{R}^n \to \mathbb{R}$ and $g_i(\bar{x}_i): \mathbb{R}^i \to \mathbb{R}^m$, $i=1,\ldots,n$, are Borel measurable, continuous, and satisfy $f(0)=0$, $g_i(0)=0$ for all $t \ge 0$, $i=1,\ldots,n$. Here we define $\bar{g}_i^T(\bar{x}_i) = [g_1^T(\bar{x}_1),\ldots,g_i^T(\bar{x}_i)]$.

Obviously, the Lyapunov function approach in the traditional backstepping design is no longer effective for the nonlinear systems and the virtual control signals are too hard to design. To overcome the difficulty of system (1.1), Khoo et al. [23] have recently developed a backstepping design method to discuss almost surely finite-time stabilization of system (1.1). Generally speaking, the finite-time stability concerns the stability of the equilibrium which can be obtained in finite-time while the globally asymptotic stability pay attention to the behavior of the system in infinite time. Pan and Basar [24] considered the asymptotic stability in large and developed a recursive control design for a class of stochastic nonlinear systems with strict-feedbacks. Usually, it is necessary to determine the globally asymptotic stability of a nonlinear system. Indeed, it can be further applied in some other related problems, such as the filtering problems, finite-horizon estimation, and tracking control, see [25–28]. Deng and Krstic [29] systematically