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On robust and optimal imperfect information state equipartitioning for network systems ☆

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Abstract

This paper addresses some fundamental questions towards robust and optimal imperfect information state equipartitioning by means of a computational framework of a system-theoretic approach called semistable Gaussian Linear-Quadratic Consensus (LQC), which is motivated by Optimal Semistable Control (OSC). OSC deals with an optimal regulation problem for dynamical systems with unknown, nonzero set-points. In this paper, we address stochastic OSC for robust and optimal information state equipartitioning under Gaussian white noise disturbance or measurement and random distribution of initial conditions. We develop a new framework for semistable Gaussian LQC and recast the proposed problem into an alternative, constrained optimization problem. To this end, necessary and sufficient conditions to connect semistability and optimal information state equipartitioning are derived in the paper and the existence of optimal solutions to this optimization problem has been proved for network systems. To efficiently solve the proposed constrained optimization problem, we propose a convergence guaranteed numerical algorithm. The rigorous convergence proof of this algorithm is presented and simulation results show the efficacy of the proposed approach.

1. Introduction

The information state equipartitioning principle for network systems has been widely used in many different engineering system applications such as communication protocol design for

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wireless networks [1], distributed Kalman filtering for distributed sensor fusion [2], continuous-time Markovian jump processes for stochastic models of chemical reactions [3], swarm intelligence algorithm design for complex system optimization problems [4], optimal resource allocation for network systems [5], consensus, synchronization and control of complex networks [6], and system thermodynamics [7], to cite but a few. This principle means that for each agent of the system, with the information interaction between its neighbors, the information state of each agent can be synchronized to the same value [8]. The main challenging problem for information state equipartitioning is to design proper state equipartitioning protocols to achieve agreement in different scenarios. Hence, there are a lot of research efforts towards this challenge. For instance, the authors in [1] develop random gossip algorithms for the wireless network system to achieve information state equipartitioning, whereas the communication link between every pair of two agents is randomly selected. Moreover, [9] proposes a gossip algorithm via a quantized communication, which is quite general for many practical applications. At the same time, ergodic theory emerges as a new tool for information state equipartitioning design and has a close relationship with the consensus problem [10,11]. Using this theory, one can develop a unified framework for designing both deterministic and stochastic information state equipartitioning protocols, and hence, leading to a much more general result.

Although the primary focus of the information state equipartitioning problem is on the convergence issue as well as the stability issue of the proposed information state equipartitioning protocols, the performance issue such as optimality properties of these algorithms has started gaining attention in the literature [1,12–18]. Optimality here refers to minimization of certain cost functional for coordination algorithms subject to convergence/stability and network connectivity constraints. In particular, the authors in [12,1] consider the eigenvalue optimization of linear time-invariant and time-varying averaging algorithms, respectively. Later, a least squares minimization is taken into account in [13] to minimize the cost associated with the trajectories of the linear averaging algorithm. The authors in [14] use a static optimization method of treating the state of other agents as constant when evaluating the minimum value of each individual cost. Reference [15] uses dynamic programming to solve an optimal leader–follower problem while Reference [17] takes the decomposition idea to formulate a Linear-Quadratic-Regulator (LQR) optimal information state equipartitioning problem into a linear matrix inequality problem. Similar LQR-type optimization problems are also considered in [16,18] with a local information–interaction-type cost functional.

However, there is a fundamental problem remaining in these optimal information state equipartitioning protocols developed in the literature. While convergence or stability of these optimal information state equipartitioning protocols has been guaranteed for network systems, a robustness question for these optimal information state equipartitioning problems naturally arises: *Are these optimal information state equipartitioning protocols developed in the literature robust to exogenous disturbance or uncertainty*? Here we consider the robustness issue of information state equipartitioning protocols with respect to imperfect information. To our best knowledge, this important question has been largely ignored though considerable attention has been concentrated on the convergence performance of many optimal information state equipartitioning protocols in the literature and the answer to this question remains widely open. Hence, the first fundamental contribution in this paper is to give a surprisingly *negative* answer to this question. As a consequence, our counter-intuitive conclusion unveils a serious flaw for many LQR-type optimal information state equipartitioning protocols developed in the literature and poses a serious doubt for applications of these results in practice since nonrobust information state equipartitioning protocols are practically useless in real-world system engineering problems.