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Journal of the Franklin Institute 353 (2016) 1742–1758

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Complex function projective synchronization in drive-response complex-variable dynamical networks with coupling time delays

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Received 22 January 2015; received in revised form 30 November 2015; accepted 7 February 2016
Available online 26 February 2016

Abstract

This paper investigates complex function projective synchronization (CFPS) in drive-response complex-variable dynamical networks, which means that the complex-variable chaotic systems in networks may evolve in different directions with a time-varying intersection angle in a complex plane and can be synchronized up to a complex scaling function. Hybrid feedback control schemes are proposed to achieve CFPS in drive-response complex-variable networks with both constant coupling delay and time-varying coupling delay. Based on Lyapunov stability theory and the linear matrix inequality, some sufficient conditions are derived by adopting an appropriate Lyapunov–krasovskii energy function. Finally, the numerical simulation experiments are employed to verify the correctness and the effectiveness of the proposed method and further reveal that the synchronization is independent of the coupling delay.

1. Introduction

For the recent decades, there has been increasing interest in the study of synchronization phenomena, and many different types of synchronization in the coupled chaotic systems or complex dynamical networks have been investigated, such as complete synchronization [1], phase synchronization [2], lag synchronization [3], cluster synchronization [4], generalized synchronization [5], and projective synchronization [6]. Amongst all the types of synchronization, projective

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http://dx.doi.org/10.1016/j.jfranklin.2016.02.003

synchronization (PS) is one of the most noticeable problems and has been extensively studied because of its proportionality between the synchronized dynamical states with a scaling factor and its use to achieve fast communication [8]. To develop and extend projective synchronization, modified projective synchronization (MPS) [9], function projective synchronization (FPS) [10,11], generalized matrix projective synchronization(GMPS) [12] and generalized function projective synchronization (GFPS) [13] are later investigated by researchers.

All the above researches only considered the synchronization of coupled dynamical systems with real variables [1–13]. But in reality, many real systems can be only modeled by dynamical systems with complex variables. For example, rotating fluids and detuned laser [14,15] can be described and simulated by the complex Lorenz system as follows [16]:

$$\begin{cases} \dot{x} = \sigma(y-x) \\ \dot{y} = rx - xz - ay \\ \dot{z} = \frac{1}{2}(\overline{x}y + x\overline{y}) - bz \end{cases} \tag{1}$$

where x, y are complex variables and z is a real variable. The parameters r and a are complex numbers defined by $r = r_1 + jr_2$, $a = 1 - j\delta$, and $\sigma, b, r_1, r_2, \delta$ are real and positive. Therefore, researches on dynamics and control of complex chaotic systems have achieved some results [17].

Since not only the complex chaotic systems have great importance and broad applications, but also complex variables which double the number of variables can increase the contents and security of the transmitted information, synchronization between complex chaotic systems has caused much concern over the last few decades [18]. In Ref. [19], complex Chen and Lü systems are introduced and then the global synchronization of coupled identical systems are investigated. Later, by using active control Mahmoud et al. [20] studied the chaos synchronization of two different complex Chen and Lü systems. In Ref. [21], Adaptive anti-lag synchronization of two identical or non-identical hyperchaotic complex nonlinear systems with uncertain parameters was well discussed. As for projective synchronization, Mahmoud et al. [22] studied modified projective lag synchronization of two nonidentical hyperchaotic complex nonlinear systems. Liu et al. [23] proposed adaptive modified function projective synchronization of general uncertain chaotic complex systems. Luo et al. [24] investigated hybrid modified function projective synchronization of two different dimensional complex nonlinear systems with parameters identification.

In the above works of projective synchronization, the scaling factors which the complex chaotic systems are synchronized up to are real numbers, real matrices or real valued functions. That is to say the driver-response systems change in the same direction or inverse direction. However, in order to make the transmitted signals have stronger anti-attack ability and anti-translated capability, scaling factors of projective synchronization have been extended to the complex domain, which means the system states can change in different directions. According to these, some related works have been given. In Ref. [25], the concept of complex projective synchronization in coupled chaotic complex systems is introduced. In Ref. [26], Mahmoud et al. studied complex modified projective synchronization of the drive-response complex systems. Liu et al. [27] achieved complex function projective synchronization of complex chaotic system and applied it in secure communication.

It is clear that the above researches only considered complex projective synchronization between the systems with complex-variables. But a small number of papers discussed complex projective synchronization in or between the networks with complex-variables. For instance, Wu et al. [28] introduced the concept of complex projective synchronization in drive-response