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# Asynchronous event-triggered control of multi-agent systems with Sigma–Delta quantizer and packet losses

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**Abstract**

This paper investigates the consensus problem for event-triggered multi-agent systems (MAS) subject to external disturbances, time delays and packet losses as well as Sigma–Delta ($\Sigma\Delta$) quantizer who has a finite number of quantization level simultaneously. Firstly, we transform this problem into a robust $H_\infty$ control by defining an appropriate controlled output. Secondly, we give a criteria to judge the consensusability of asynchronous event-triggered MAS and present a sufficient condition in terms of matrix inequalities to get the state feedback controller's parameters. Moreover, the maximal allowable number of successive packet losses is also given, and the algorithm designed here has made considerable contributions to save communication resources. Finally, numerical examples are given to show the effectiveness of the proposed consensus protocol.


## 1. Introduction

During the last decades, the distributed or decentralized cooperative control of MAS has witnessed an increasing interest, which is partly due to their broad applications in many fields, including biology, physics, robotics, vehicles and control engineering [1–7]. In these fields, one of the most important and fundamental issues is consensus control of MAS, which is the key to find control protocols such that all of the agents can reach an agreement and has been extensively studied in the past few years with many profound results [8–10], and the references herein.

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In real MAS, topological structure, time delays, packet dropouts and external disturbances are the mainly sources of unconsensusability [11] due to the limited communication capacity of systems. Therefore, it is significant to develop methods that more effectively use the limited bandwidth available for transmitting state information. To overcome this problem, quantization has recently become an active research topic [12–14]. Quantizers are divided into static quantizers (memoryless quantizers) and dynamic quantizers (with memory). Whether continuous-time or discrete-time networked system, the previous researches on the quantized consensus protocol commonly used static quantizer, such as logarithmic quantizer or uniform quantizer [15,16]. As we all know that logarithmic quantizer is commonly used in many control settings, but it requires an infinite number of quantization levels. And due to the static quantizer is memoryless, which has steady-state error, quantized consensus may not be obtained eventually. Typically, a quantizer with a very low bit rate is needed, and recent focus has been on minimizing information rate for feedback [17]. However, practical control design based on the minimum information rate is difficult and how to merge quantization with a finite number of levels is always a difficulty in the study of quantification.

Based on this reason, we discuss the consensus problem using Sigma–Delta ($\Sigma\Delta$) quantizer. $\Sigma\Delta$ quantizer is an effective tool which is designed to quantize the limited bandwidth signal by Inose and Yasuda in 1963 in the literature [18]. Since then, as a practical quantizer which quantizes over sampling limited bandwidth signals, the application of $\Sigma\Delta$ quantizer began to mature. Being a method of dynamic quantization, $\Sigma\Delta$ quantizer makes up for the presence of the steady-state error of static quantizers. Besides, compared with the infinite bit of quantization information of a logarithmic quantizer, $\Sigma\Delta$ quantizer can obtain high precision resolution ratio using finite bit of information communication, which possesses a better performance and saves resources [19]. Therefore, $\Sigma\Delta$ quantizer can deal with the long term problem of using finite information rate in a simple way, which has more practical significance and is expected to obtain commendable consensus results.

Meanwhile, when updating some dynamic parameters in the control systems, the time-driven [20] and the event-driven [21–25] fashion are widely used. The former gives the traditional way that all the state signals are transmitted through communication networks. Clearly, it may be conservative to guarantee the stability of a network or a convergence of an algorithm, while the worst cases are that the former control method will lead to sending of many unnecessary signals to a network, which may lead to a huge wastage of communication bandwidth. In view of this, it seems that an event-triggered type is more favourable in MAS [26–32], where signals whether or not to be transmitted to the controller are determined by certain events that are triggered depending on some rules. The rules provide a useful way of determining when the quantized signals are sent out. Then, the system can adjust the quantization rates adaptively in a certain way dependent on the current state information of the system. Meng and Chen [29] studied an event based average consensus problem for multiple integrators over fixed, or switching, undirected and connected network topologies. In [30], Zhu et al. studied the event-triggered control of MAS under the general linear model.

Most of the event-triggered control protocols for MAS are synchronous, where a unified state error threshold is set up for all agents. While, there are another kind of event-triggered protocols for MAS called asynchronous algorithms. For MAS, agents only need to communicate with their neighbours. Set a state error threshold based on the neighbours for each agent, and when the state error reaches the threshold, the agent triggers the update operation alone, and to ensure that all the agents can achieve consensus under the conditions of every agent triggered individually, which is called asynchronous event-triggered consensus algorithm for MAS. The asynchronous event-triggered control can avoid the problems of synchronous control to some extent, reduce the