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Stability analysis for interval time-varying delay systems based on time-varying bound integral method ☆

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Abstract

This paper addresses the new stability analysis method for systems with interval time-varying delay. By taking single-integral and double-integral terms with time-varying bound into consideration, a new Lyapunov–Krasovskii functional is defined. Then reciprocally convex approach and some transformations are used to estimate the derivative of the constructed functional less conservatively, and as a result, some new stability criteria are obtained in terms of the quadratic convex combination, which are less conservative and have less decision variables. Two well-known examples are also given to illustrate the advantage of the main results.

1. Introduction

Time-varying delays are often encountered in practical control systems, and may result in poor performances and even instability, so the stability of time-varying delay systems has been studied extensively in the last years. In order to reduce the conservatism of the stability results, many approaches were developed. The main efforts were concentrated on two aspect, one is the

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construction methods of L–K functional, such as delay-fraction functional [1,2,20], augmented functional [3,4], functional with matrices depending on the time-delays [5], functional including quadratic terms multiplied by a higher degree scalar function [6,7], L–K functional with triple-integral terms [8] and functional with quadruple-integral terms [9]; the other is analysis methods estimating the derivatives of functionals, such as integral inequality lemma [10–12], the improved bounding technique [13–15], free weighting matrix method [16,3,21], the convex analysis [17,18], quadratic convex analysis [7], reciprocally convex approach [19] and second-order reciprocally convex approach [22]. As we known, to reduce the conservatism of the stability condition, the construction of L–K functional is the first key point. If the information of time-varying delay is fully used in the L–K functional, the stability result will be less conservative. From existing papers, we can find that, to the constructed functionals, the single-integral term with time-varying bound and the integral terms with constant bound were always used, but the double-integral term with time-varying bound was not used, especially when time-varying delay $h(t)$ belongs to interval $[h_1, h_2]$.

Motivated by above-discussion, we study the stability of systems with interval time-varying delay in this paper. Firstly, a novel L–K functional is introduced, which contains single-integral and double-integral terms with time-varying bound. Then, in order to reduce computational burden, the derivative of L–K functional is estimated by using reciprocally convex approach and new transformation, and as a result, more general and less conservative stability conditions are obtained in terms of the quadratic convex combination. Finally, numerical simulations are used to show the effectiveness of the main results.

Throughout the note, the used notations are standard. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is a set of $n\times m$ real matrix, A^T is the transpose of A, $P>0$ ($P<0$) means symmetric positive (negative) definite matrix, and $*$ in the matrix denotes the symmetric element, I is the identity matrix of appropriate dimensions, $x_t = x(t+\theta), \theta \in [-h, 0]$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulations

Consider the following interval time-delay system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t-h(t)) \\ x(t) &= \phi(t), \quad t \in [-h, 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; The initial condition $\phi(t)$ is a continuously differentiable vector-valued function; $A, A_1 \in \mathbf{R}^{n\times n}$ are known real constant matrices; $h(t)$ is the time-varying delay satisfying

$$h_1 \le h(t) \le h_2 \tag{2}$$

$$\dot{h}(t) \le d \le \infty \tag{3}$$

where h_1, h_2, d are constants.

To obtain the main results, the following lemmas are needed.

Lemma 1 (Kim [7]). *For any symmetric matrix X_0, X_1, X_2 and a vector ξ, let*

$$f(\alpha) = \xi^T X_0 \xi + \alpha \xi^T X_1 \xi + \alpha^2 \xi^T X_2 \xi$$

with $X_2 > 0$. Then $\forall \alpha \in [\alpha_1, \alpha_2]$, we have $f(\alpha_1) < 0$ and $f(\alpha_2) < 0 \Longrightarrow f(\alpha) < 0$.