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Synchronization of biological neural network systems with stochastic perturbations and time delays ☆

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Abstract

With advances in biochemistry, molecular biology, and neurochemistry there has been impressive progress in the understanding of the molecular properties of anesthetic agents. However, despite these advances, we still do not understand how anesthetic agents affect the properties of neurons that translate into the induction of general anesthesia at the macroscopic level. There is extensive experimental verification that collections of neurons may function as oscillators and the synchronization of oscillators may play a key role in the transmission of information within the central nervous system. This may be particularly relevant to understand the mechanism of action for general anesthesia. In this paper, we develop a stochastic synaptic drive firing rate model for an excitatory and inhibitory cortical neuronal network in the face of system time delays and stochastic input disturbances. In addition, we provide sufficient conditions for global asymptotic and exponential mean-square synchronization for this model.

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1. Introduction

Numerous complex large-scale dynamical networks often demonstrate a degree of synchronization. System synchronization typically involves coordination of events that allows a dynamical system to operate in unison resulting in system self-organization. The onset of synchronization in populations of coupled dynamical networks has been studied for various complex networks including network models for mathematical biology, statistical physics, kinetic theory, bifurcation theory, as well as plasma physics [1]. Synchronization of firing neural oscillator populations using probabilistic analysis has also been addressed in the neuroscience literature [2]. One of the most important questions in neuroscience is how do neurons, or collections of neurons, communicate. In other words, what is the neural code? There is extensive experimental verification that collections of neurons may function as oscillators [3–5] and the synchronization of oscillators may play a key role in the transmission of information within the central nervous system. This may be particularly relevant to understand the mechanism of action for general anesthesia [6].

It has been known for a long time that general anesthesia has profound effects on the spectrum of oscillations in the electroencephalograph [7,8]. More recently, the authors in [9] have suggested that thalamocortical circuits function as neural pacemakers and that alterations in the thalamic oscillations are associated with the induction of general anesthesia. Furthermore, it is well known that anesthetic drugs frequently induce epileptiform activity as part of the progression to the state of unconsciousness [10].

Multiple lines of evidence indicate that anesthetic agents impact neural oscillators. In addition, epileptiform activity implies synchronization of oscillators. This leads to the possibility that synchronization of these oscillators is involved in the transition to the anesthetic state. In this paper, we extend the synaptic drive model of a network of biological neurons developed in [6] to investigate the conditions that would lead to synchronization of neural oscillators. In particular, we develop an excitatory and inhibitory synaptic drive firing rate model with time-varying delays and stochastic input uncertainty, and its global synchronization is investigated. The system uncertainty model involves a Markov process wherein stochastic integration is interpreted in the sense of Itô.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n \times m}$ denotes the set of $n \times m$ real matrices, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, $\|\cdot\|$ denotes the Euclidean vector norm, and $\mathcal{C}([-\tau, 0], \mathbb{R}^n)$ with $\tau > 0$ denotes a family of continuous vector-valued functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with topology of uniform convergence and designated operator norm $\|\psi\| = \sup_{-\tau \le \theta \le 0} \|\psi(\theta)\|$ for $\psi \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$. We write I or I_n for the $n \times n$ identity matrix, $\mathbf{1}_n$ for the $n \times 1$ ones vector, $\mathrm{tr}(\cdot)$ for the trace operator, $\mathrm{rank}\, A$ for the rank of the matrix A, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of a Hermitian matrix A, $\ker(A)$ for the kernel (nullspace) of the matrix A, $\mathrm{span}(\mathbf{1}_n)$ for the span of $\mathbf{1}_n$, $\mathbb{E}[\cdot]$ for the expectation operator for a given probability space, and $\mathbb{E}[X|Y]$ for the conditional expectation of X with respect to Y.

2. Biological neural networks

The fundamental building block of the central nervous system, the *neuron*, can be divided into three functionally distinct parts, namely, the *dendrites*, *soma* (or cell body), and *axon*. The dendrites play the role of input devices that collect signals from other neurons and