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Stabilization irrespective of bounds of uncertain variations for linear uncertain systems with delays

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Abstract

In this study, the stabilizability irrespective of the bounds of uncertain parameters and time delays is investigated for linear uncertain delay systems. For uncertain systems without delays, a linear time-varying or time-invariant uncertain system has been shown to be stabilizable independent of the bounds of uncertain variations if and only if the system has a particular geometric configuration called an antisymmetric stepwise configuration (ASC) or a generalized antisymmetric stepwise configuration (GASC), respectively. In this study, fundamental approaches to investigating the stabilizability of delay systems with specific uncertainty structures such as ASCs or GASCs are presented. For a class of 3-dimensional systems, it is shown here that if a linear time-varying or time-invariant uncertain delay system has an ASC or a GASC, respectively, then the system can be stabilized, however large the given bounds of delays and uncertain parameters might be.

1. Introduction

It is useful to classify the existing results on the stabilization of uncertain systems into two categories. The first category includes results in [1–6] that provide stabilizability conditions that are dependent on the bounds of uncertain parameters. The second category includes results in [7–9] that provide stabilizability conditions that are independent of the bounds of uncertain parameters, but dependent on the locations of uncertain entries in system matrices. The stabilizability conditions in the second category can be verified simply by examining the

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positions of uncertain entries in given system matrices. Once a system satisfies the stabilizability conditions, a stabilizing controller can be constructed, however large the given bounds of uncertain variations might be. We can redesign the controller to improve robustness simply by modifying the design parameter when the uncertain parameters exceed the upper bounds given beforehand. In this study, we specifically address the second category.

In the second category, the stabilizability conditions have a particular geometric configuration with respect to the permissible positions of uncertain entries in system matrices. Using the concept of the antisymmetric stepwise configuration (ASC) or generalized antisymmetric stepwise configuration (GASC), it was proved in [7,8] that a linear time-varying or time-invariant uncertain system without delays is stabilizable regardless of the given bounds of uncertain variations if and only if the system has an ASC or a GASC, respectively.

For a class of delay systems, it was shown in [9] that if a linear time-varying uncertain delay system has a triangular ASC, then the system is stabilizable irrespective of the given bounds of uncertain parameters and time delays. However, the method in [9] is inapplicable to a class of systems with all admissible ASCs, because its applicability is limited to the triangular configurations, as shown in Fig. 1 in [9]. The stabilization problem was reduced in [9] for finding the proper means of choosing the order structures of the eigenvalues corresponding to given uncertainty configurations. A unified method of choosing the proper order of the eigenvalues is only applicable to a class of systems with triangular ASCs. The difficulty in solving this problem for all possible ASCs is the large variety of classifications of the means of choosing the proper eigenvalues. Hence, here, we focus on only 3-dimensional systems for simplicity.

So far, we have discussed the stabilization problem of time-varying uncertain delay systems. However, the stabilization problem of time-invariant uncertain delay systems has not yet been addressed in the second category. The previous approach [9] is useless for developing the stabilizability conditions of systems with GASCs. Therefore, in this study, we provide a novel stabilizability criterion for time-invariant systems in a general dimensional setting. Nevertheless, the stabilization problem of time-invariant systems is also reduced to finding the proper order structure of the eigenvalues similar to that of time-varying systems. For this reason, the difficulty in overcoming the diversity of classifications of the means of choosing the proper eigenvalues remains in this problem. Hence, we also focus on only 3 dimensions for time-invariant systems.

The objective of this study is to provide fundamental approaches to investigating the stabilizability of linear time-varying or time-invariant uncertain delay systems with ASCs or GASCs, respectively. In this paper, we present a novel stabilizability criterion for time-invariant uncertain delay systems in a general dimensional setting. Furthermore, we show here that 3-dimensional linear time-varying or time-invariant uncertain delay systems with all possible ASCs or GASCs are stabilizable, respectively, however large the given bounds of uncertain parameters and time delays might be. This means the stabilizability conditions of linear uncertain delay systems are not degraded by the existence of delays that are allowed to take arbitrarily large values. The intrinsic nature of high-dimensional structures can be considered to be adequately reflected in that of 3-dimensional structures.

Notation: Throughout this paper, the notation $\star$ always denotes the location of an uncertain entry. For $A, B \in \mathbb{R}^{n\times m}$, every inequality between A and B, such as $A > B$, indicates that it is satisfied componentwise. If $A \in \mathbb{R}^{n\times m}$ satisfies $A \geq 0$, A is called a non-negative matrix. The determinant and the transpose of $A \in \mathbb{R}^{n\times n}$ are denoted by $\det(A)$ and A', respectively. For $A = (a_{ij}) \in \mathbb{R}^{n\times m}$, $|A|$ denotes a matrix with $|a_{ij}|$ as its (i,j) entries. Let $diag\{\cdots\}$ and $I \in \mathbb{R}^{n\times n}$ denote a diagonal matrix and an identity matrix, respectively. A real square matrix, wherein all its off-diagonal entries are non-positive, is called an M-matrix if it is non-singular, and its inverse