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Journal of the Franklin Institute 351 (2014) 1596–1614

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Stabilization of the stochastic jump diffusion systems by state-feedback control

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Received 26 April 2013; received in revised form 21 September 2013; accepted 11 December 2013
Available online 19 December 2013

Abstract

This paper addresses the stabilization of stochastic jump diffusion system in both almost sure and mean square sense by state-feedback control. We find conditions under which the solutions to the class of jump-diffusion process are mean square exponentially stable and almost sure exponentially stable. We investigate the stabilization of the stochastic jump diffusion systems by applying the state-feedback controllers not only in the drift term, but also in jump diffusion terms. Meanwhile our theory is generalized to cope with the uncertainty of system parameters. All the results are expressed in terms of linear matrix inequalities (LMIs), which are easy to be checked in a MATLAB Toolbox.

1. Introduction

During the last decades, stochastic jump diffusion systems have become increasingly popular tools to describe the real world [1]. The jump component can capture event-driven uncertainties, such as corporate defaults, operational failures, or insured events. Indeed, such stochastic differential equations are finding a considerable range of applications, including

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0016-0032/$32.00
http://dx.doi.org/10.1016/j.jfranklin.2013.12.009

physical sciences [2, Chapter 4], biology, engineering [3], financial economics in nonlinear signal processing [4, Chapter 10], and stochastic resonance [5]. In financial and actuarial modeling and other areas of application, such jump diffusions are often used to describe the dynamics of various state variables. In finance these may represent, for instance, asset prices, credit ratings, stock indices, interest rates, exchange rates or commodity prices.

Each stochastic jump diffusion, consisting of the continuous part and discontinuous jump, can be written as a linear combination of time t, a Brownian motion $w(t)$ and a pure jump process. When the jump part of the system vanishes, one has a stochastic diffusion, known as stochastic differential equation (SDE). When the Brownian motion in the diffusion is also missing, the system reduces to the deterministic system, known as ordinary differential equation (ODE) [6]. This work is concerned with the stochastic jump diffusion systems. The Poisson jump process makes the formulation more versatile with a wider range of applications. In addition, it makes the study more difficult. So our targeted system is much more difficult to deal with than the SDE and ODE.

One of the important issues in the study of stochastic jump diffusion systems is automatic control, with consequent emphasis being placed on the asymptotic analysis of stability. Unlike the deterministic systems, stochastic systems may refer to the different stability concepts, such as asymptotic stability in probability, almost sure (exponential) stability, and mean square (exponential) stability, etc. Stability analysis of stochastic jump diffusion systems has attract much attention. Applebaum and Siakalli [7] discussed the asymptotic stability properties of stochastic differential equations driven by Lévy noise. Yin and Xi [6] investigated the stability of a class of switching jump-diffusion process. Zong et al. [8] discussed stability and stochastic stabilization of numerical solutions of a class of regime-switching jump diffusion systems. In [9], Applebaum and Siakalli showed that the Lévy noise can also be used to stabilize the unstable system almost surely. Bao and Yuan [10] studied the stabilization of partial differential equations by Lévy noise. These studies have not taken the structure of feedback control into account. For the stochastic systems without Poisson jump, there is an intensive literature and we mention, for example, [11–17] and the references therein. In particular, the two articles [17,16] are more typical in the feedback control of stochastic diffusion systems. Li and Blankenship [18] studied the stabilization of unstable systems only driven by Poisson jump by feedback controls. There are a few recent studies dealing with the control design problem of jump diffusion systems, for example, Øksendal and Sulem [1] proposed the stochastic control in economics and finance model.

This paper is concerned with the almost sure exponential stabilization and the mean square exponential stabilization of stochastic jump diffusion equations by feedback control. Assume that we are given an unstable linear jump diffusion equation

$$dx(t) = Ax(t)\,dt + \sum_{i=1}^{d} B^i x(t)\,dw_t^i + Cx(t-)\,dN_t, \tag{1.1}$$

with $x(0) = x_0 \in \mathbb{R}^n$, where $x(t) \in \mathbb{R}^n$ denotes the state vector, A, B^i and C are matrices in $\mathbb{R}^{n\times n}$, $w_t = (w_t^1, w_t^2, \ldots, w_t^d)^T$ is an d-dimensional Brownian motion and N_t is a Poisson process (independent of w_t) with the jump intensity λ. The two stochastic processes w_t and N_t are adapted to a filtration $\mathfrak{F}_t$. This can be the natural filtration $\mathfrak{F}_t = \sigma\{w_s, N_s, s \le t\}$.

In order to stabilize the unstable diffusion system (1.1), we may need to restrict the control not only in drift term but also in the jump diffusion terms. Because that: (1) There are lots of systems which cannot be stabilized if the control is restricted only in shift or jump diffusion parts, which can be shown similarly with the Example 1 in [17, Chapter 4]. (2) The theory developed in this