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New delay-dependent bounded real lemmas of polytopic uncertain singular Markov jump systems with time delays

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Abstract

This paper addresses the $\mathcal{H}_\infty$ performance analysis problem for singular Markov jump delayed systems with polyhedral parameter uncertainties. By introducing two useful inequalities (see Lemma 2), some new bounded real lemmas (BRLs) are obtained based on a novel parameter-dependent Lyapunov functional. The presented BRLs can guarantee that the considered system is stochastically admissible and satisfies a prescribed $\mathcal{H}_\infty$ performance level. Two numerical examples are provided to demonstrate the improvement of the proposed method over the existing methods.

1. Introduction

Over the past several decades, there has been a growing interest in singular systems, which are also known as descriptor systems, generalized state-space systems, implicit systems, or differential-algebraic systems. The popularity of singular systems stems from their ability to provide better description of the behavior of many practical systems than regular systems (state-space systems). Such systems can be found in many practical systems, power systems [5], constrained robot systems [10], chemical processes [13], economic systems [33], to name a few. Consequently, the study of singular systems is of paramount importance, and has been received considerable attention. In particular, the problems of stability analysis and control synthesis have

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been investigated thoroughly from the control community [13,33]. It is worth pointing out that, in comparison with the regular systems, the study on the stability analysis and control synthesis of singular systems is more complicated. The reason is that the resulting closed-loop systems are required to be not only stable, but also regular and impulse-free (for continuous-time singular systems) or causal (for discrete-time singular systems) simultaneously.

On another research front, Markov jump systems (MJSs) are a class of important stochastic hybrid systems, where information exchange between the modes is governed by a continuous time finite-state Markov chain [4,9,12,22,24,41]. The MJSs have been proved to be a very useful tool to analyze and model random abrupt changes in parameters and structures of many physical systems, such as manufacturing systems, telecommunications systems, and powers systems [3,4,11,40]. The past few decades have witnessed a considerable resurgence of interest in the study of MJSs, and a lot of control issues for MJSs have been investigated in the literature, see e.g. [7,18,20,27,28] and the references therein. On the other hand, time delays exist unavoidably in many practical control systems. The existence of time delays may degrade the performance, or even lead to instability of control systems if the time delays are large enough [1,23]. As a result, much effort has been devoted to the study of delayed systems in order to improve robustness and stability against time delays [2,16,39]. For the above reasons, it is not surprising that considerable attention has been focused on singular Markov jump delayed systems (SMJDSs), and many significant results have been reported [29–32].

Among these issues, the problem of $\mathcal{H}_\infty$ control synthesis for SMJDSs has received much attention in the past few years, where the bounded real lemma (BRL) plays an important role in solving $\mathcal{H}_\infty$ control synthesis problem [8,19,21,34,44]. Therefore, there have been several bounded real lemmas for singular Markov jump systems in recent years [25,33]. Furthermore, when time delays appear in a singular Markov jump system, a delay-dependent bounded real lemma for a class of discrete-time SMJDSs was proposed in [14] by using the discrete Jensen inequality, while the delay-dependent BRLs for continue-time SMJDSs were established in [26,31]. However, it is important to note that the delay-dependent BRLs derived in [26,31] still involve some conservatism, and may be improved by using some new methods. Moreover, the polytopic uncertainties were also not studied in [26,31]. These motivate the following research.

In this paper, the delay-dependent $\mathcal{H}_\infty$ performance analysis problem for a class of continue-time uncertain SMJDSs, where the parameter uncertainties are assumed to belong to a given convex polytope. The aim is to exploit new analysis methods to achieve novel delay-dependent BRLs. For this purpose, a new lemma (see Lemma 2) is introduced based on an extended Wirtinger inequality and the reciprocally convex approach. By using this lemma and a novel parameter-dependent Markovian switched Lyapunov functional, we present some new delay-dependent BRLs, which ensure that the considered system is not only stochastically admissible, but also satisfies a prescribed $\mathcal{H}_\infty$ performance level. Finally, we provide two numerical examples to demonstrate the effectiveness and the reduced conservatism of our proposed method.

Notation: Throughout this paper, for symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite); I is the identity matrix with appropriate dimension. The notation M^T represents the transpose of the matrix M; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$; $\mathcal{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; $|\cdot|$ refers to the Euclidean vector norm; $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm, while $\|\cdot\|_{E_2}$ denotes the norm in $\mathcal{L}_2((\Omega,\mathcal{F},\mathcal{P}),[0,\infty))$, where $(\Omega,\mathcal{F},\mathcal{P})$ is a probability space; Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$.