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Stability of impulsive delayed linear differential systems with delayed impulses

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Abstract

In this paper, uniform stability problem of impulsive delayed differential systems with delayed impulses is investigated. By means of the Lyapunov function method combined with Razumikhin technique and classic analysis methods, the criteria of uniform stability, uniform asymptotic stability and exponential stability for impulsive delay systems with delayed impulses are obtained. The numerical examples are given to illustrate the effectiveness of the proposed conclusions.

1. Introduction

Over the past several decades, impulsive dynamical systems have attracted increasing interest because they provide a natural framework for mathematical modelling in the fields of science and engineering, such as biology, physics, control technology, industrial robotics, and communication engineering (see, for example, [1–12]). Studying impulsive delayed systems is very important for its inevitable in many practical problems. Stability is one of the most important issues in the study of impulsive delay differential equations. In recent years, there have been many research works in the literature on impulsive delay differential equations (see, for example,

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[13–43]) and many stability criteria for impulsive delay systems have been proposed. However, most research results on impulsive delay equations do not consider delayed impulses, which are mainly due to some theoretical difficulties in this kind of systems.

Lyapunov–Razumikhin method is one of the most useful ways in dealing with the stability of impulsive delayed differential systems, see [12–30] and the references cited therein. Recently, making use of Lyapunov functional and analysis technique, the stability issue has been studied in [12–28, 32–34] for some impulsive control systems with time delay. Several criteria on uniformly stability, uniformly asymptotic stability and exponential stability were established, and those results show that a system can be stabilized by impulses even if it contains an unstable system matrix. In [13,21], the authors proved the criteria on uniform stability and exponential stability for linear impulsive delayed systems based on Lyapunov function technique. Liu and Ballinger considered the uniform asymptotic stability of impulsive delay differential equations in [32]. Ho and Sun [14] investigated the uniform stability and the uniform asymptotic stability for the Takagi–Sugeno fuzzy delay systems. A unified synchronization criterion for impulsive dynamical networks was investigated in [28]. In these papers, the impulses were usually assumed to take the form $\Delta x(t_k) = x(t_k^+) - x(t_k^-) = B_k x(t_k^-)$, which indicates the state 'jump' at the impulse times t_k, and there have been few available results on stability analysis of impulsive time-delay systems with delayed impulses [17]. However, in the transmission of the impulse information, input delays are often encountered [43]. Based on this fact, it is necessary to deeply study the stability of impulsive delayed systems with delayed impulses. In recent years, by means of the fundamental matrix of differential systems and some restrictions on delays, Liu et al. [24] considered the stability problem of some particular classes of impulsive delayed systems with delayed impulses. Uniform stability for nonlinear systems with delayed impulses was studied in [16]. In [25], Khadra et al. used exponential estimates to investigate the asymptotic stability for delay-free autonomous impulsive control systems with delayed impulses. Making use of Lyapunov function method combined with Razumikhin technique, Chen et al. [17–19,29,30] studied the exponential stability for the time-delay nonlinear impulsive differential systems, nonlinear singularly perturbed systems and Takagi–Sugeno fuzzy systems with delayed impulses. Although several results of impulsive delayed systems with delayed impulses have been obtained in [17–19,24–30], some restrictions must be illustrated to get these results. Moreover, most of these results were discussed on exponential stability. Therefore, the existing research on stability of impulsive delayed systems with delayed impulses may still be insufficient.

In this paper, we will further discuss the stability for the impulsive linear and nonlinear delayed systems with delayed impulses. By ways of Lyapunov functions, differential inequalities and Razumikhin technique, we will obtain the uniform stability criteria, uniform asymptotic stability criteria and exponential stability criteria of delayed systems with delayed impulses respectively. Three numerical examples will be given to illustrate the effectiveness of our results. From these examples, it can be seen that proper delayed impulse does not affect the stability, but it may change the convergence speed and process of impulsive system.

The remainder of this paper is organized as follows. In Section 2, we introduce some basic notation and definitions. We establish several stability criteria for linear and nonlinear impulsive delayed systems with delayed impulses in Section 3. Finally, conclusions are given in Section 4.

2. Preliminaries

The following notations and definitions will be used in this paper. Let R be the set of real numbers, R_+ the set of nonnegative real numbers and R^n the space of n-dimensional column vectors $x = \mathrm{col}(x_1, x_2, \ldots, x_n)$ with the Euclidean norm. I is the identity matrix of order n.

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