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# Stabilization and passification of distributed-order fractional linear systems using methods of preservation

Guillermo Fernández-Anaya<sup>a</sup>, José-Job Flores-Godoy<sup>a,\*</sup>, Armando-Fabian Lugo-Peñaloza<sup>a</sup>, Rodrígo Muñoz-Vega<sup>b</sup>

<sup>a</sup>Departamento de Física y Matemáticas, Universidad Iberoamericana, México, D. F., Mexico <sup>b</sup>Universidad Autónoma de la Ciudad de México, Centro Histórico, México, D. F., Mexico

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#### **Abstract**

This paper studies the stabilization and passification of a class of distributed-order linear time-invariant systems, by using methods of preservation in the frequency domain. Results about preservation of stability and passivity of classical linear time-invariant systems are extended to one more general family of matrix functions. Based on these results, a new approach to the problems of stabilization and passification of distributed-order linear time-invariant systems is presented. Also a result that extends the known techniques for pole placement of classical linear time-invariant systems to the new class of distributed-order linear time-invariant systems is given. Examples are given to show the validity of theoretical results.

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#### 1. Introduction

Fractional calculus allows for integral and differential operators of arbitrary order. As a consequence, the order of the fractional derivative can be a function of time, space or some other variable [1,2]. Systems with fractional order derivatives are very significant, not only in physics, mechanics, chemistry, bioengineering, robotics, etc., where it is shown that just the fractional order of derivatives of several practical classes of problems is a better natural description of the studied phenomenon, see e.g., [3–5]. Also, these types of systems have already a very important contribution in the control area, signal processing, see e.g., [6–9].

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<sup>\*</sup>Corresponding author. Tel.: +52 55 59504000; fax: +52 55 59504275.

*E-mail addresses:* guillermo.fernadez@ibero.mx (G. Fernández-Anaya), job.flores@ibero.mx, job.flores@uia. mx (Flores-Godoy), armando.lugo@correo.uia.mx (Lugo-Peñaloza), rodrigo.munoz@uacm.edu.mx (R. Muñoz-Vega).

Fractional derivatives of distributed order, a generalization of fractional order derivatives, were first discussed by Caputo [10–12]. Distributed order systems have recently been a subject of interest for researchers in diverse fields. For example, the existence and uniqueness of solutions for such systems are discussed in [13], where numerical approximations are also discussed. Explicit strong solutions for these systems are provided in [14], along with stochastic analogues for distributed order time-fractional diffusion equations on bounded domains, while [15] is dedicated to for the discretization of distributed order operators.

In the past few years stabilization and passification methods have appeared for linear time-invariant (LTI) systems [16–20], methods which are useful when addressing various problems in classical control theory. On the other hand, stabilization (and more so passification) has been neglected in linear systems of distributed order, even when these latter systems have lately gained importance, as attested by the recent publication of a monograph on the subject [21], and the appearance of papers that treats the distributed order systems as generalizations of the fractional order ones [13,22]. Moreover, recently in [15] the impulse response of the distributed order integrator/differentiator and its asymptotic property is derived by using the complex path integral. Then, they present a technique to perform the discretization of the above distributed order integrator/differentiator. In [23], the analytical impulse response of a fractional-order distributed parameter low-pass filter is derived. The asymptotic properties of the impulse responses are obtained too. In [24], a physical experimental study of temperature-dependent variable-order fractional integrator and differentiator is introduced. Also, frequency domain preservation methods are currently applied to different topics in linear fractional order systems, including among others, stability of order preservation [25], discretization methods [26] and numerical methods for the simulation of such systems [27]. Yet, for linear systems of distributed order there are, up to our best knowledge, virtually no preservation methods at all.

Moreover, while a methodology for the preservation of stabilization and passification has been developed in recent years for LTI systems [28–32], no analogue exists for linear fractional order systems, much less for distributed order ones.

Motivated by the above open problems for linear fractional distributed order systems, in this paper we develop several results on preservation of stabilization and passification of a family linear fractional distributed order systems, which generalize the class of systems frequently reported in the literature [33–35,8], using preservation methods in the frequency domain. We extend recent results on stability of linear fractional distributed order systems in [22]. These results allow us to extend some classical methods for stabilization and passivation of LTI systems, to a new family of linear fractional distributed order systems, which includes among other the distributed order linear time invariant systems (DOLTIS) in [21]. The paper is organized as follows. In Section 2, we formulate the problem and present some necessary preliminaries and also give some relevant results of preservation in the frequency domain. In Section 3, results for the problems of stabilization and passification of a large class of distributed-order fractional linear systems and stabilization of a more general class of systems via pole placement by static output feedback with restrictions are presented. Numerical examples are finally presented in Section 4 to illustrate some application of our theoretical results, which is followed by conclusions in Section 5.

#### 2. Problem formulation, preliminaries and basic results

Fractional order derivatives, which generalize the integer order derivative, have been defined in several different ways. One of the most commonly used definitions is the so-called Caputo derivative, which we will use all along this paper.

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