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Robust synchronization of a class of chaotic networks ☆

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Abstract

This paper studies synchronization of a dynamical complex network consisting of nodes being generalized Lorenz chaotic systems and connections created with transmitted synchronizing signals. The focus is on the robustness of the network synchronization with respect to its topology. The robustness is analyzed theoretically for the case of two nodes with two-sided (bidirectional) connections, and numerically for various cases with large numbers of nodes. It is shown that, unless a certain minimal coherent topology is present in the network, synchronization is always preserved. While for a minimal network where synchronization is global, the resulting synchrony reduces to semi-global if redundant connections are added.

1. Introduction

The research topic of complex networks has revoked considerable interest in the past few years. Examples of complex networks in interest include the Internet, World Wide Web, food webs, electric power grids, metabolic networks, and biological neural networks, among many others [1,2]. Traditionally, complex networks were studied via random graph theory, introduced by Erdös and Renyí [3], which have been extended to a wide extent in the last decade.

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This paper studies the synchronization phenomenon of dynamical complex networks (DCN), where all nodes are identical chaotic systems (but usually with different parameters and/or initial conditions). Compared to existing results, there are two novel features in our new approach. First, nonlinear synchronizing connections between nodes are allowed; secondly, a directed graph as a model for DCN is considered, in contrast to the general studies where only linear coupling and undirected networks are discussed [4–7]. Note that a general approach to the local synchronization of chaotic systems for any linear coupling scheme was described in [8]. The objective here is to study the synchronizability of the network when some nodes establish or lose certain connections. This notion is referred to as *structural robustness of DCN synchronization*. The motivation comes from the consideration that in a network numerous participants try to synchronize to each other for some reason (e.g. for chaotic secure communication [9,10]), while some participants may connect to or disconnect from some of their partners under certain conditions, but these should not damage the overall synchrony of the network. It will be shown that with an increasing number of connections, synchronization is only semi-global, and it may become even worse as the number of connections continue to increase, in the sense that very high gains (coupling strengths) are needed to maintain the synchrony of the whole network.

This paper aims to study DCN consisting of the so-called generalized Lorenz system (GLS) [11,12]. It was already shown in [12] that two GLS's in master–slave configuration can be synchronized using a single scalar connection. Further, some bidirectionally coupled synchronization results on GLS were obtained in [13], using nonscalar, but linear connections. The present paper will continue the initial study presented in [14], considering both scalar nonlinear bidirectional connection between two GLS's, with mathematical proof of the convergence, and a study of more complex network topologies of up to eight GLS nodes. As an example of good synchronization properties even for larger number of nodes and connections, Fig. 9 shows some possible topologies of eight-node networks and Fig. 10 illustrates their error dynamics. While mathematical proofs for more complex cases are unrealistic, numerical studies show quite interesting behavior, e.g. increasing the numbers of nodes and connections usually leads to increasing sensitivity with respect to initial synchronization errors.

The rest of the paper is organized as follows. Some notions related to the DCN and the synchronization problem are introduced in the next section. The DCN of GLS is then discussed in Section 3, together with theoretical analysis on the DCN with two coupled nodes for its synchronization. Numerical simulations on the GLS-based DCN, with 3, 4 and 8 nodes, are presented in Section 4. Finally, conclusions are given in the last section.

2. Synchronization of dynamical complex networks

Consider a DCN of N identical nonlinear nodes, with each node being a chaotic system, described by

$$\dot{\eta}^i = f(\eta^i) + \sum_{j=1}^{N} c_{ji}\phi(\eta^i, h(\eta^j), L), \tag{1}$$

where $\eta^i = (\eta_1, \eta_2, \ldots, \eta_n,)^\top \in R^n$ is the state vector of node i, $i = 1, \ldots, N$, $L = (l_1, l_2, \ldots, l_n)^\top$ is the vector of coupling gains, $h(\cdot)$ is a scalar synchronizing output of each system, ϕ is nonlinear coupling with $\phi(\eta, h(\eta), L) \equiv 0\ \forall \eta, L$ and $C = (c_{ij})_{i,j=1,\ldots,n}$ is the adjacency matrix that has no loops, i.e. $i \neq j$. Here, c_{ij} is not always equal to c_{ji}, because the graph is directed, i.e. the adjacency