ARTICLE IN PRESS

ELSEVIER

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of the Franklin Institute ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

On higher-order boundary value problems by using differential transformation method with convolution terms

Adem Kiliçman*, Omer Altun

Department of Mathematics and Institute for Mathematical Research, University Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia

Received 13 October 2011; received in revised form 3 July 2012; accepted 14 August 2012

Abstract

In this paper, we study higher-order boundary value problems for higher-order nonlinear differential equations having convolution terms. We extend and prove some theorems for nonlinear differential equations by using the differential transform method.

1. Introduction

The concept of differential transform (one-dimensional) was first proposed and applied to solve linear and nonlinear initial value problems in electric circuit analysis by Zhou, see [37]. This method constructs an analytical solution in the form of a polynomial. The differential transform method (DTM) is a transformation technique based on the Taylor series expansion and is a useful tool to obtain analytical solutions of the differential equations.

The well known advantage of DTM is its simplicity and accuracy in calculations and also wide range of applications. In the literature, there are several studies where the DTM is used in order to deal with linear and nonlinear initial value problems. For example, Chiou and Tzeng [8] applied the Taylor transform to solve nonlinear vibration problems, Abdel-Halim Hassan [11] adopted the DTM to solve some eigenvalue problems, and related to the aeroelasticity problems, see [7]. In the DTM, certain transformation rules are applied and the governing differential

*Corresponding author.
E-mail addresses: akilicman@putra.upm.edu.my, kilicman@yahoo.com (A. Kiliçman).

0016-0032/$32.00 © 2012 The Franklin Institute. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jfranklin.2012.08.010

equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions and the solution of these algebraic equations gives the desired solution of the problem. It is different from the high-order Taylor series method since Taylor series method requires symbolic computation which is expensive for large orders. The DTM is an iterative procedure for obtaining analytic Taylor series solutions of linear and non-linear ordinary or partial differential equations, see [15,16].

Ayaz in [4] performed two and three-dimensional differential transformation method to find exact solutions of linear and nonlinear partial differential equation. Results are compared to decomposition method and DTM has less computational effort. After that, Erturk and Momani in [10] presented numerical solution by comparing the differential transformation method (DTM) and Adomian decomposition method (ADM) for solving linear and nonlinear fourth-order boundary value problems and proved that DTM is very accurate and efficient in numerical solution.

Recently, Arikoğlu and Ozkol solved fractional differential equations by using differential transform method. They applied fractional differential equations to various types of problems such as the Bagley–Torvik, Ricatti and composite fractional oscillation equations, see [2].

In this study, we consider some partial differential equations by using the differential transformation method with convolutions term. Further, we also propose a new method to solve the partial differential equations having singularity by using the convolution. In this new method when the operator has some singularities then we multiply the partial differential operator with continuously differential functions by using the convolution in order to remove the singularity. Then the differential transform method will be applied to the new partial differential equations that might also have some fractional order.

Now if we have the partial differential equations and which are in the form of

$$P(D)u = f(x)$$

then if we multiply the differential operator with a function by using the convolution, that is,

$$(Q(x) * P(D))u = f(x).$$

In particular if we consider $Q(x) = \delta_n(x)$ that is the delta sequence converges to Dirac delta δ that new equations is equivalent to the original equations, see [20–25]. By using the derivatives of the δ distribution then we obtain a strange but very useful following statements:

$$f = \delta * f, \quad f' = \delta' * f, \quad f'' = \delta'' * f.$$

Thus if the n-th order linear differential equation has constant coefficients, we may write it as $f * x = b$ by introducing the Dirac delta distribution.

The purpose of this study is to show the use of the convolution in order to convert the differential equation to integro-differential equation and compute the solutions explicitly by using the differential transform method.

2. Differential transformation method

Suppose that the function $y(x)$ is continuously differentiable in the interval $(x_0 - r, x_0 + r)$ for $r > 0$ then we have the following definition.